OCR Further Pure 1

Complex Numbers

Section 2: Equations and geometrical representation

Exercise

1. Given that $z_1 = 3 + 2i$ and $z_2 = 4 – i$ represent z_1, z_2, $z_1 + z_2$ and $z_1 – z_2$ on an Argand diagram.

2. Given that $z = 2 + i$ show on an Argand diagram z, z^*, iz, and iz^*. Write down the transformation that describes the relationship between
   (i) z and z^*
   (ii) z and iz

3. Write in the form $f(z) = 0$, where $f(z)$ is a polynomial of degree 4 with real coefficients, the equation having $(3 + i)$ and $(1 + 3i)$ as two of its roots.

4. Find the real root of the equation $z^3 + z + 10 = 0$ given that one complex root is $1 – 2i$.

5. Given that $1 + i$ is a root of the equation $z^3 – 2z + k = 0$ find the value of k and the other two roots.

6. Show that $z = -1 + i$ is a root of the equation $z^4 – 2z^3 – z^2 + 2z + 10 = 0$ and find the other roots.

7. Given that one root of the equation $z^3 + az + b = 0$ is $p + qi$, where a and b are real and b is not zero, prove that
   (i) $3p^2 – q^2 = -a$
   (ii) $2p(p^2 + q^2) = b$
   (iii) p is a root of the equation $8x^3 + 2ax – b = 0$

8. By writing $(a + bi)^2 = 3 – 4i$ find values for a and b and hence find the square roots of $3 – 4i$.