

DIFFERENTIAL EQUATIONS - additional notes

An equation in which at least one term contains $\frac{dy}{dx}$ is called a differential equation.

This type, in particular is called a first order differential equation.

For example, $3y\frac{dy}{dx} = 5x^2$ is a differential equation.

The equation represents a relationship between 2 variables but can be expressed in a form that does not contain a differential coefficient.

This process is known as finding *the general solution* to a differential equation.

Consider $3y\frac{dy}{dx} = 5x^2$

Integrating both sides wrt x ;

Note : We need only to introduce the constant 'c' to one side.

Generally ;

If $f(y)\frac{dy}{dx} = g(x)$ then $\int f(y)dy = \int g(x)dx + c$

6 Quick questions...find the general solutions to

1. $\frac{dy}{dx} = 3x^3$

3. $\frac{ds}{dt} = u + at$

5. $\frac{dy}{dt} = \frac{t^3 - 3t + 4}{t^3}$

2. $\frac{dy}{dx} = 3ax^2$

4. $\frac{dy}{dt} = \left(1 + \frac{1}{t}\right)\left(1 - \frac{1}{t}\right)$

6. $\frac{dy}{dx} = xy$

The particular solution of a differential equation

Here we need to find the value of the constant 'c' from the general solution. This completes the relationship between the variables.

Worked example

Find the particular solution to the differential equation

$$y \frac{dy}{dx} = x + xy^2 \quad x = 2 \quad \text{and} \quad y = 0$$

Firstly, separate the variables;

$$\frac{y}{1+y^2} dy = x dx$$

Integrate both sides

Use
Integration by
recognition

$$\int \frac{y}{1+y^2} dy = \int x dx$$

$$= \frac{1}{2} \ln(1+y^2) = \frac{1}{2} x^2 + C \quad x = 2 \quad \text{and} \quad y = 0$$

$$\Rightarrow \quad C = -2$$

Therefore the particular solution reads

$$\ln(1+y^2) = x^2 - 4 \quad (\text{double all terms})$$

Questions on particular solutions.

1. $\frac{dy}{dx} = \sin 3x \quad x = 0, \quad y = 1$

3. $y^2 \frac{dy}{dx} = x \quad x = 2, \quad y = 3$

2. $\frac{dy}{dx} = \frac{1}{2+x} \quad x = -1, \quad y = 1$

4. $\tan x \frac{dy}{dx} = y \quad x = \frac{\pi}{2}, \quad y = 1$

Answers

1. $3y = 4 - \cos 3x$

2. $y = 1 + \ln(2+x)$

3. $2y^3 = 3x^2 + 42$

4. $y = \sin x$