Integration of functions in the form $(ax+b)^n$

Consider the derivative of $(3x + 2)^4$

$\frac{dy}{dx} = 12(3x+2)^3$ (Using the chain rule)

This must mean that the reverse idea will be $\int (3x+2)^3\,dx = \frac{1}{4\times 3}(3x+4)^4$

This 3 will can 'cancel out' the derivative of the inner function

This leads to a general result : $\int (ax+b)^n\,dx = \frac{1}{n+1}(ax+b)^{n+1} \times \frac{1}{a}$

Or more simply $\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1}$

Questions and examples

1. Integrate the following functions.

(a) $(4x + 1)^5$ (b) $2(1-2x)^3$ (c) $(3+\frac{1}{2}x)^4$

(d) $(3x-2)^{-3}$ (e) $\frac{1}{\sqrt{2x+1}}$ (f) $\frac{3}{2(5x+6)^3}$

Remember that integration calculates area under a curve so we need to employ limits and apply definite integration to these functions (See OCR text EX 6B, page 247 and 248)

2. Find the value of the following integral

$\int_2^3 \frac{2}{\sqrt{x+4}}dx$ Begin by re-writing as =>

=

=

= 0.79 (2sf)

We need to apply integration to exponentials and reciprocal graphs.

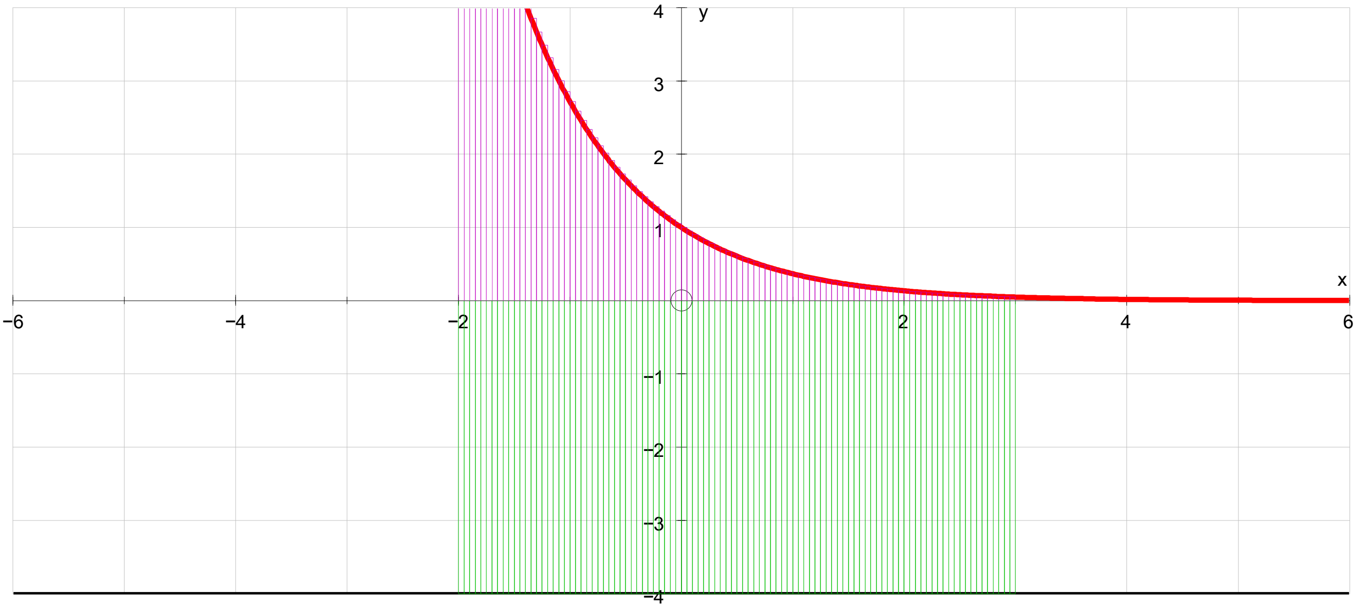
Remember that

$$\int e^x dx = e^x + c$$ and also $$\int \frac{1}{x} dx = \ln|x| + c$$

MAKE SURE YOU ARE CONFIDENT ANSWERING QUESTIONS FROM Chapter 12 (A, B & C)

Example

* Find the area bound by the curve $y = e^{-3x}$ and the line $y = -4$ between
x = – 2 and
x = 3. Give your answer to 2 d.p.



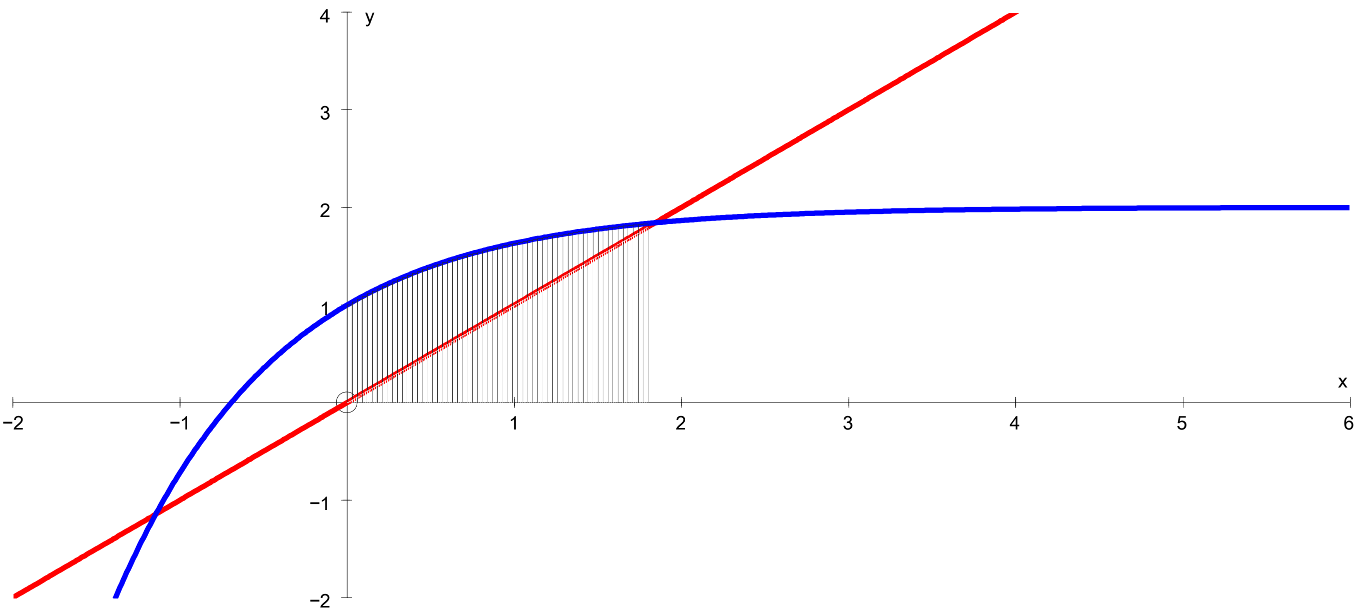
*Your answer should be 154.48 units^2.

Work your solution here

Example

The diagram shows sketches of the graphs of $y = 2 - e^{-x}$ and $y = x$.
These graphs intersect at $x = a$ where $a > 0$.

(a) Write down an equation satisfied by a. (Do not attempt to solve the equation)
(b) Write down an integral which is equal to the area of the shaded region
(c) Use integration to show that the area is equal to $1 + a - \frac{a^2}{2}$



(a) $a = 2 - e^{-a}$

$a = 2 - \frac{1}{e^a}$

$ae^a = 2e^a - 1$

$2e^a - ae^a = 1$

$e^a(2 - a) = 1$

$\boxed{e^a = \frac{1}{2-a}}$

therefore $\int_0^a 2 - e^{-x}dx \quad - \quad \int_0^a xdx$

$= \left[2x + e^{-x}\right]_0^a - \left[\frac{x^2}{2}\right]_0^a$

$= (2a + \frac{1}{e^a} - 1) - (\frac{a^2}{2})$

$= \quad 2a + (2-a) - 1 - \frac{a^2}{2}$

$= \quad 1 + a - \frac{a^2}{2}$

Integrals of the type $\int \frac{k}{ax+b}\,dx$

From previous knowledge we know that :

$\int \frac{1}{x+3}\,dx = \ln(x+3) + c$ However given that $\frac{d}{dx}\ln(2x+3) = \frac{2}{2x+3}$

This must mean ; $\int \frac{1}{2x+3}\,dx = \frac{1}{2}\ln(2x+3) + c$

Furthermore;

$\int \frac{4}{2x+3}\,dx = \frac{4}{2}\ln(2x+3) + c = 2\ln(2x+3) + c$

Generally ; $$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b)+c$$

and

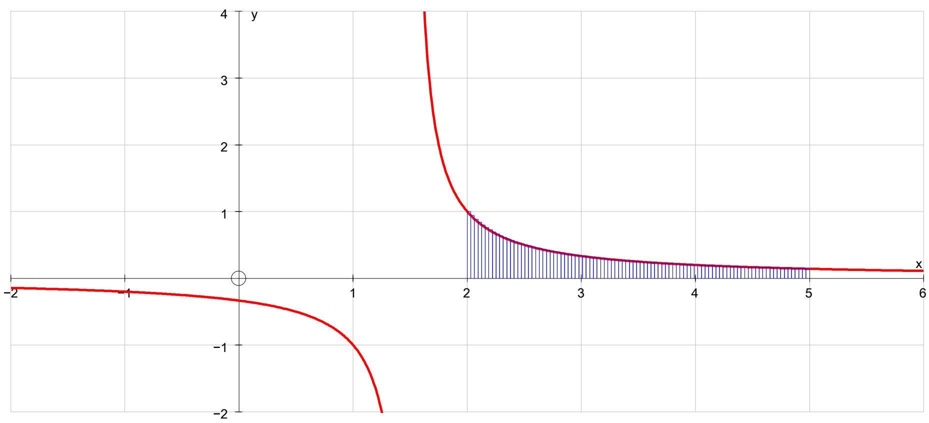
$$\int \frac{k}{ax+b}\,dx = \frac{k}{a}\ln(ax+b)+c$$

Examples to work through :

1. Find (a) $\int \frac{3}{4x-2}\,dx$ (b) $\int \frac{1}{3-x}\,dx$

2. Evaluate (a) $\int_2^5 \frac{1}{2x-3}\,dx$ (b) $\int_2^5 \frac{5}{2x+1}\,dx$

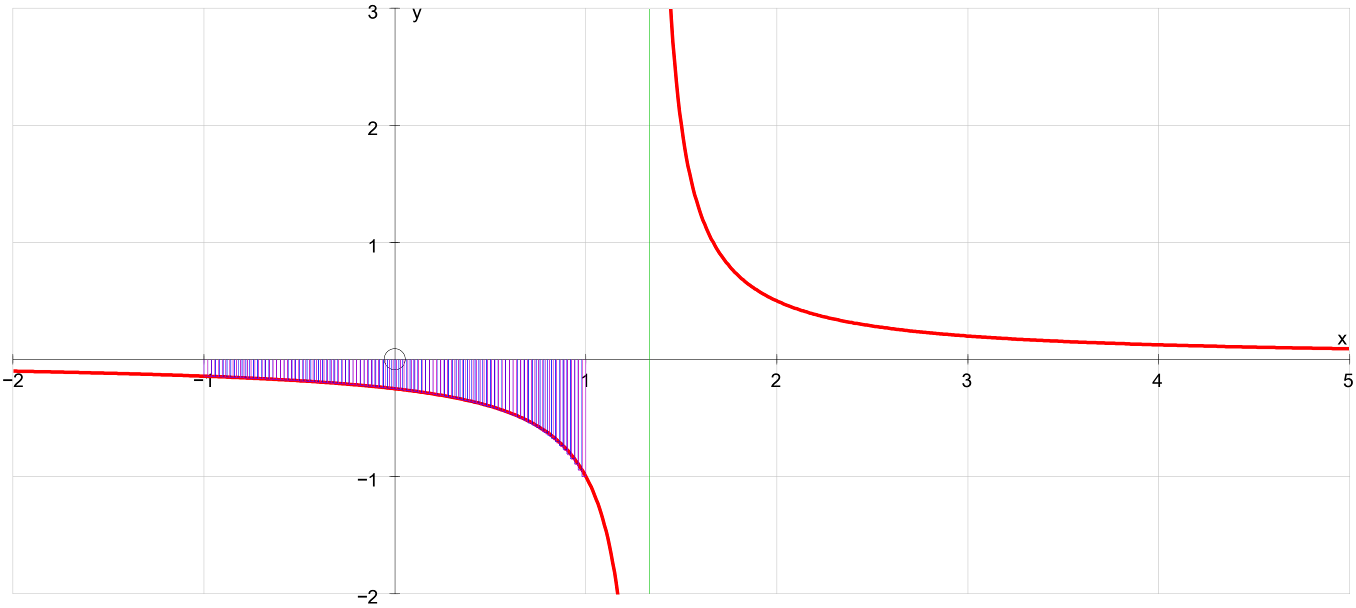
Here is the graph for 2(a) to help visualise the problem.



The problem with some of these integrals……..

Example

Evaluate the following integral $\int_{-1}^{1} \frac{1}{3x-4} dx$



$\int_{-1}^{1} \frac{1}{3x-4} dx = \left[\frac{1}{3}\ln 3x-4\right]_{-1}^{1} = \frac{1}{3}[(\ln(-1) - \ln(-7)]$

BUT $\ln(-1) and \ln(-7)$ DO NOT EXIST but the area clearly does?

Instead we need to consider ln |3x – 4 | (the modulus ignores the sign)

Therefore : $\frac{1}{3}[(\ln(1) - \ln(7)]$ = $-\frac{1}{3}\ln 7$ = __________ units^2 (2sf)