THE MODULUS FUNCTION

We have already knowledge of what | x | means . We studied the rules for convergence for a GP for example, where | x | < 1.

RECAP : | x | < 1 means − 1 < x < 1 Think about this on a number line?

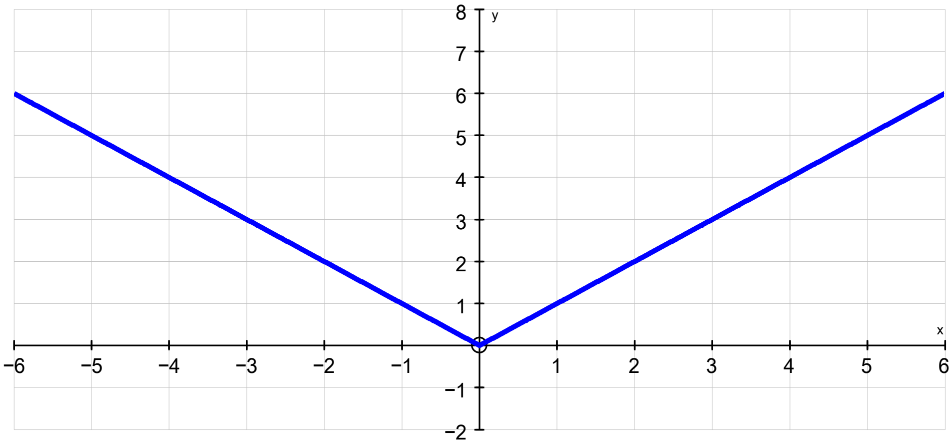
Furthermore | x | > 2 means x < − 2 or x > 2

Also | x - a| < b ⇔ − b < x - a < b

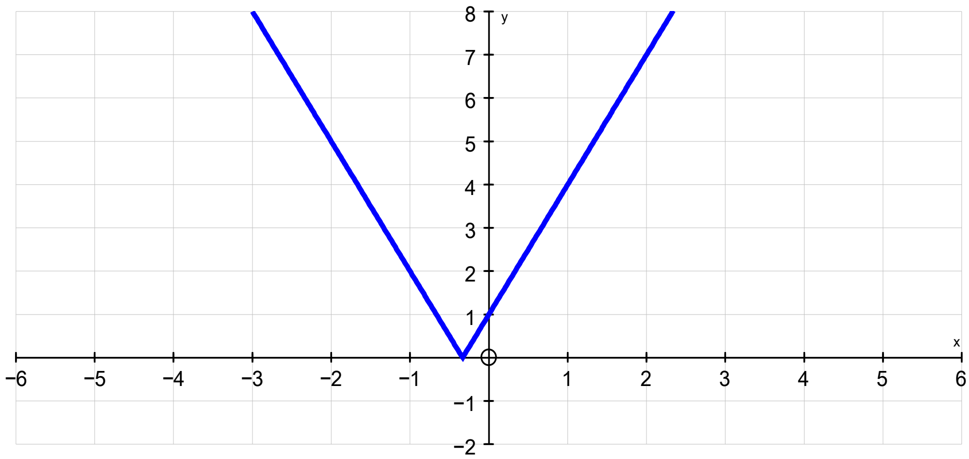
| x - a| < b ⇔ − b + a < x < b + a

Graph sketching with the modulus sign

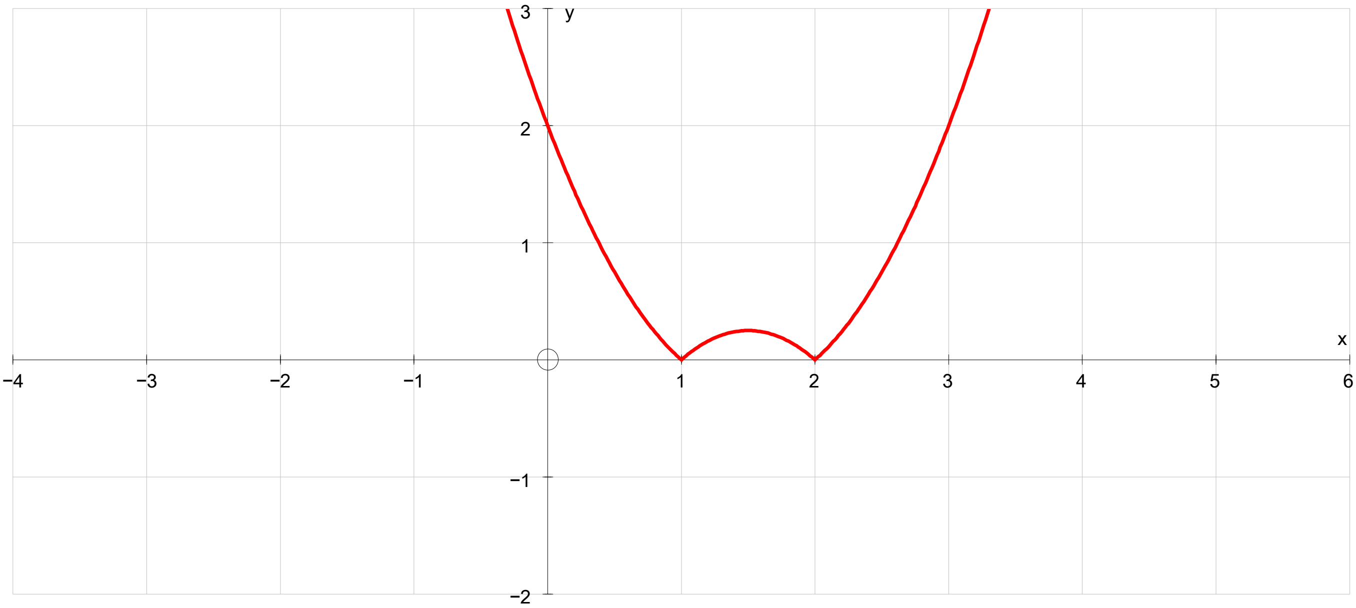
Here is the graph of y = | x |. As you can see, all negative values of y have been reflected in the x *axis.*



The same idea can be applied to other straight lines. Consider the graph of y = |3x + 1|

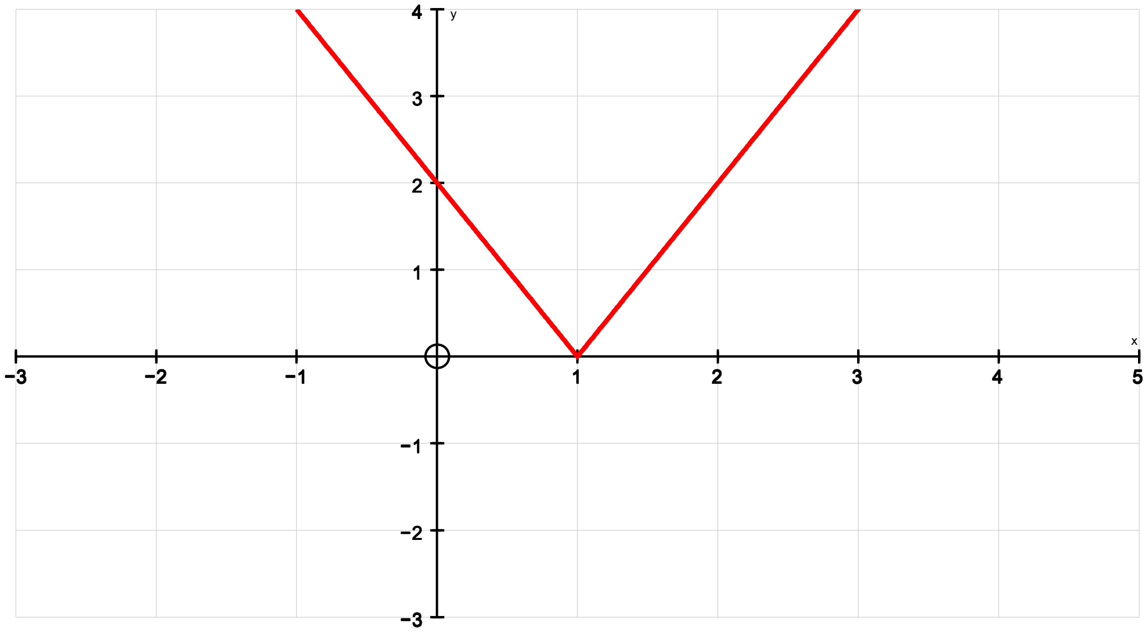


Of course the same idea could be applied to a quadratic function such as $y = |(x - 1)(x - 2)|$



By considering transformations of graphs (C1) we can accurately sketch graphs such as

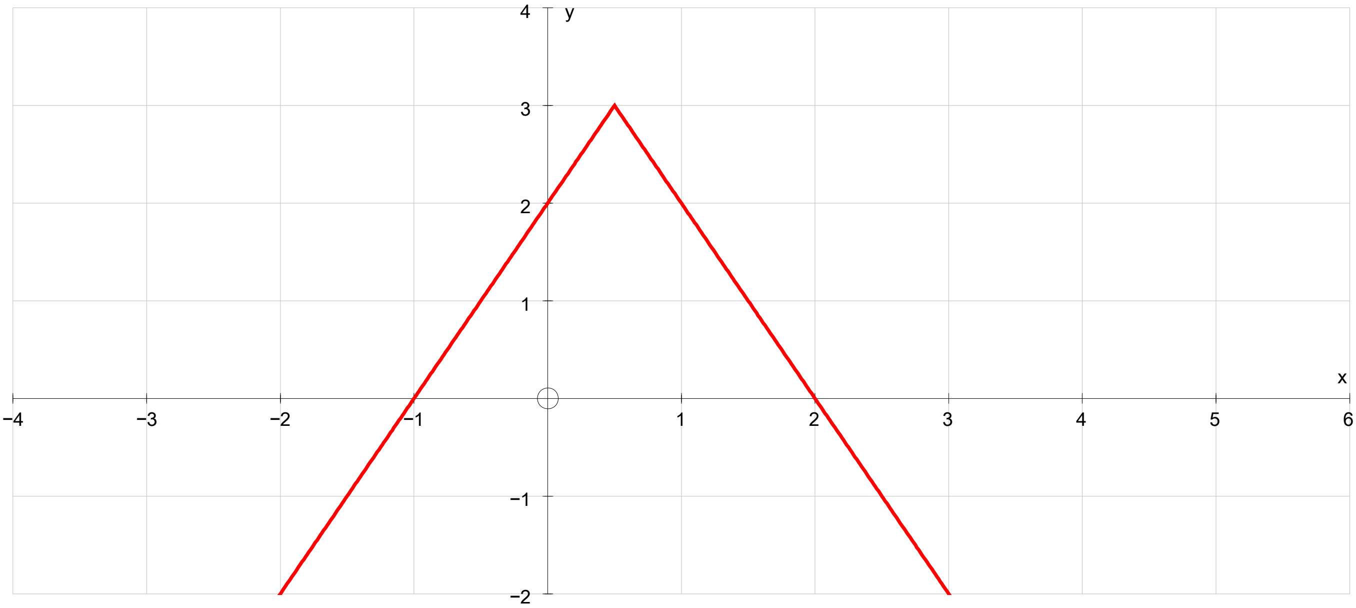
$y = 2 \mid x - 1 \mid$ **Here, the <u>gradient</u> of the line is ± 2**



$y = 3 - \mid 1 - 2x \mid$ by thinking of this graph in stages

Stage 1 > Draw the line $y = 1 - 2x$
Stage 2 > Draw the modulus $y = \mid 1 - 2x \mid$
Stage 3 > Draw the reflection in the x axis
(the transformation type $y = -f(x)$)

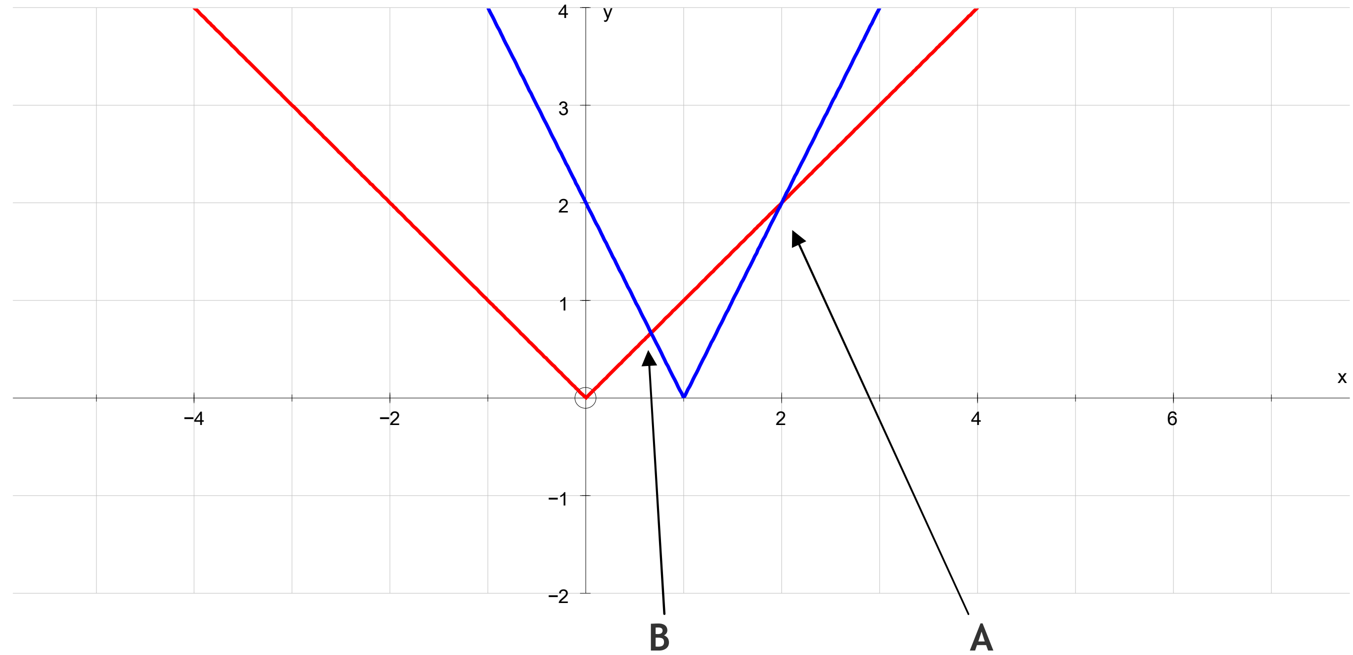
Stage 4 > Draw the graph that is translated 3 units up
Therefore looking like this



MODULUS EQUATIONS

Solve $|2x - 2| = |x|$ by using sketch graphs

We need to determine the points of intersection of the two graphs



At point A : $x = 2x - 2$ => $\underline{x = 2}$

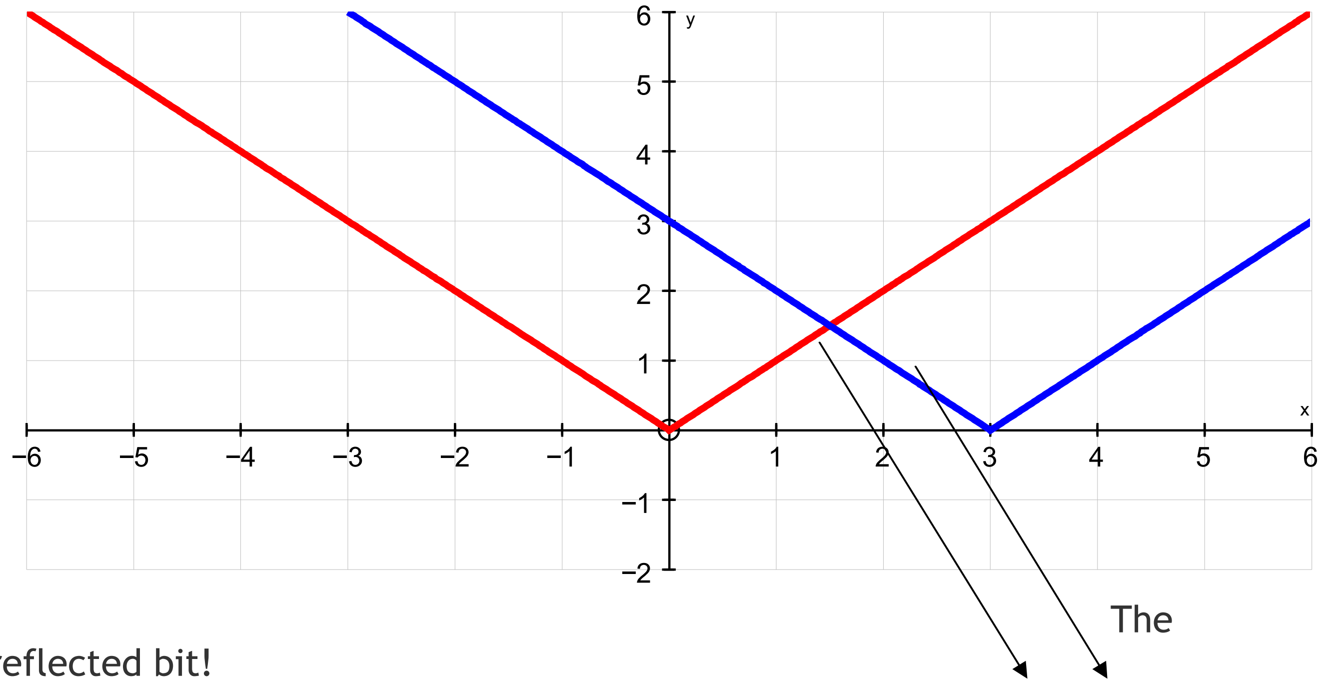
At point B : $x = -(2x - 2)$ => $3x = 2$ => $x = \frac{2}{3}$

MODULUS INEQUALITIES

These can be solved both graphically (sketching graphs to deduce intersections) or algebraically.

Example Solve $|x| < |x - 3|$

If we sketch the graphs, we need to be able to deduce which parts of the graph of $|x|$ is BELOW $|x - 3|$



The reflected bit!

From the diagram, you can see that the graphs cross where $x = -(x - 3)$

$2x = 3 \Rightarrow$

$$x = \frac{3}{2}$$

Now look for the part where $|x|$ is below $|x - 3|$

For $|x| < |x - 3| \quad \Rightarrow \quad x < \frac{3}{2}$

Alternatively,

$|x| < |x - 3|$

Square both sides

Solve inequality

This method is obviously much quicker

Exercise - Sketch the following graphs

1. $y = | 2x - 1 |$
2. $y = | x(x - 1)(x - 2) |$
3. $y = |x^2 - 1|$
4. $y = |x^2 + 1|$
5. $y = |\sin x|$
6. $y = | \ln x |$
7. $y = 2 | x + 1 |$
8. $y = | 2x + 1 | - 4$
9. $y = | x^2 - x - 20 |$
10. $y = 1 + | 2 - x^2 |$

<u>WATCH OUT FOR THIS TYPE!</u>

Sketch the graph of $y = | x + 1 | + | x - 2|$

For $x > -1$

$| x + 1 | = x + 1$ and for $x < -1$ $| x + 1 | = -(x + 1) = -x - 1$

For $x > 2$

$| x - 2 | = x - 2$ and for $x < 2$ $| x - 2 | = -(x - 2) = 2 - x$

So now we need to consider the nature of the graph at different intervals, namely when

For $x > 2$ $y = x + 1 + x - 2 = \mathbf{2x - 1}$

For $-1 < x < 2$ $y = x + 1 + 2 - x = \mathbf{3}$

For $x < -1$ $y = -x - 1 + 2 - x = \mathbf{-2x + 1}$

The graph of $y = | x + 1 | + | x - 2|$