

ROOTS FROM GRAPHS & Iterative methods

We obviously know that by solving an equation we are locating the roots - where the graph crosses the x axis.

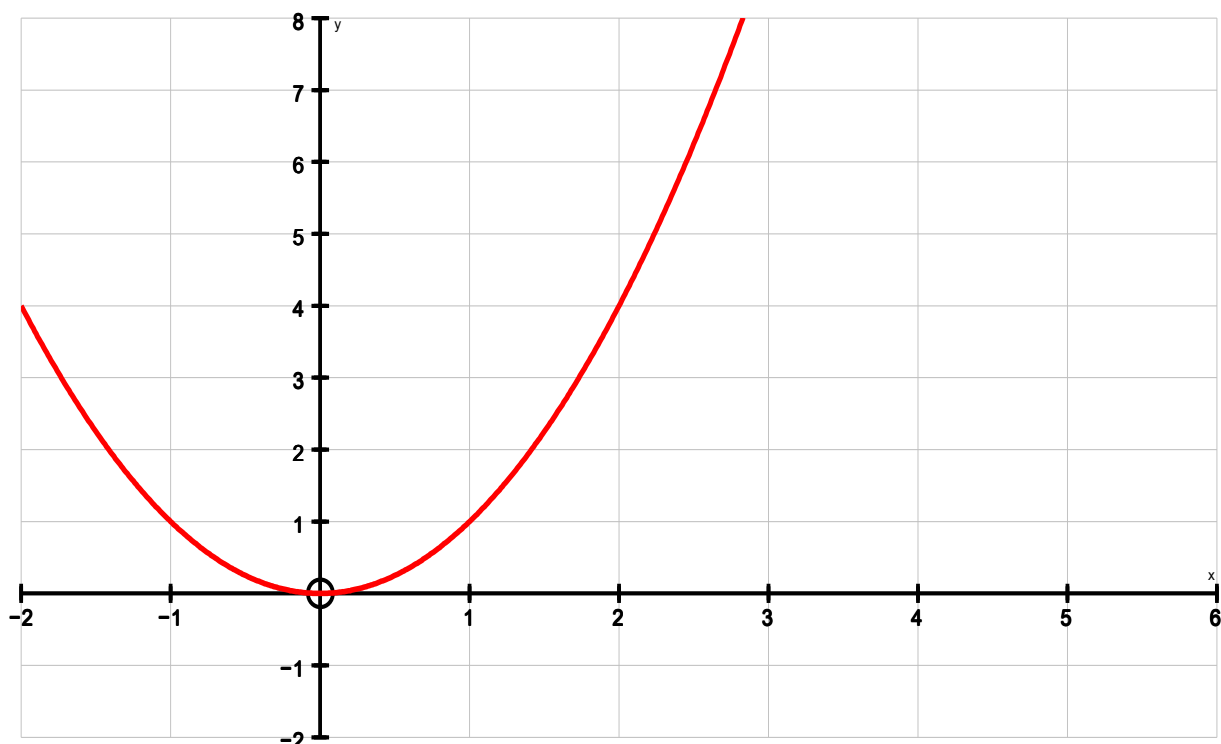
If an equation can be written in the form $f(x) = g(x)$ where $f(x)$ and $g(x)$ are both functions that could be sketched, then the number of roots and their approximate values can be found from the points of intersection of the graphs.

Take for example $x^2 - x - 3 = 0$

This equation could be re-written as $x^2 = x + 3$

- On the graph below, draw on the line $y = x + 3$
- Determine the approximate values for the roots of the equation

$$x^2 - x - 3 = 0$$



For discussion

Let $f(x) = x^2 - 3x + 2$

How could you solve

(a) $f(x) = 0$

(b) $f(x) = 2$

(c) $f(x) = x + 1$

What graph would need to be draw with $f(x)$ in order to solve

$$x^2 - 2x - 1 = 0$$

Sketch the two in the space below.

Exam style

Show that the equation $4e^{-x} + x - 4 = 0$ has only one positive root and state the integer which is nearest to this root.

- Let $f(x) = g(x)$
- Sketch both functions
- Use a suitable approach to determine the nearest integer.

Using an iterative formula

An iteration is a step by step procedure (most likely with a calculator - so have this ready!!) by which an estimate for a root can be improved.

The method works by rearranging an equation into the form

$x_{n+1} = F(x_n)$ where x_1 is the first approximate value of the root we are looking for.

(It may be worth noting here that the gradient of the curve will affect how quickly you locate the root.)

You are most likely to be given **an iterative formula** but we can build one ourselves:

Problem 1

Show how $x^2 - 5x + 2 = 0$ can be rearranged to form the iterative formula

$$x_{n+1} = 5 - \frac{2}{x_n}$$

By using $x_1 = 4$, find the positive root of the equation $x^2 - 5x + 2 = 0$ correct to 2dp.

Problem 2

Use the iteration $x_{n+1} = 4(1 - e^{-x_n})$ to find the positive root of the equation $4e^{-x} + x - 4 = 0$ giving your answer correct to 4dp.

(see previous exam style question for graph sketch)

$$x_1 = 4$$

$$x_2 =$$

$$x_3 =$$