NUMERICAL METHODS of INTEGRATION - Simpson's rule

For even number of strips n ;

$$\int_a^b f(x)dx \approx \frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right)$$

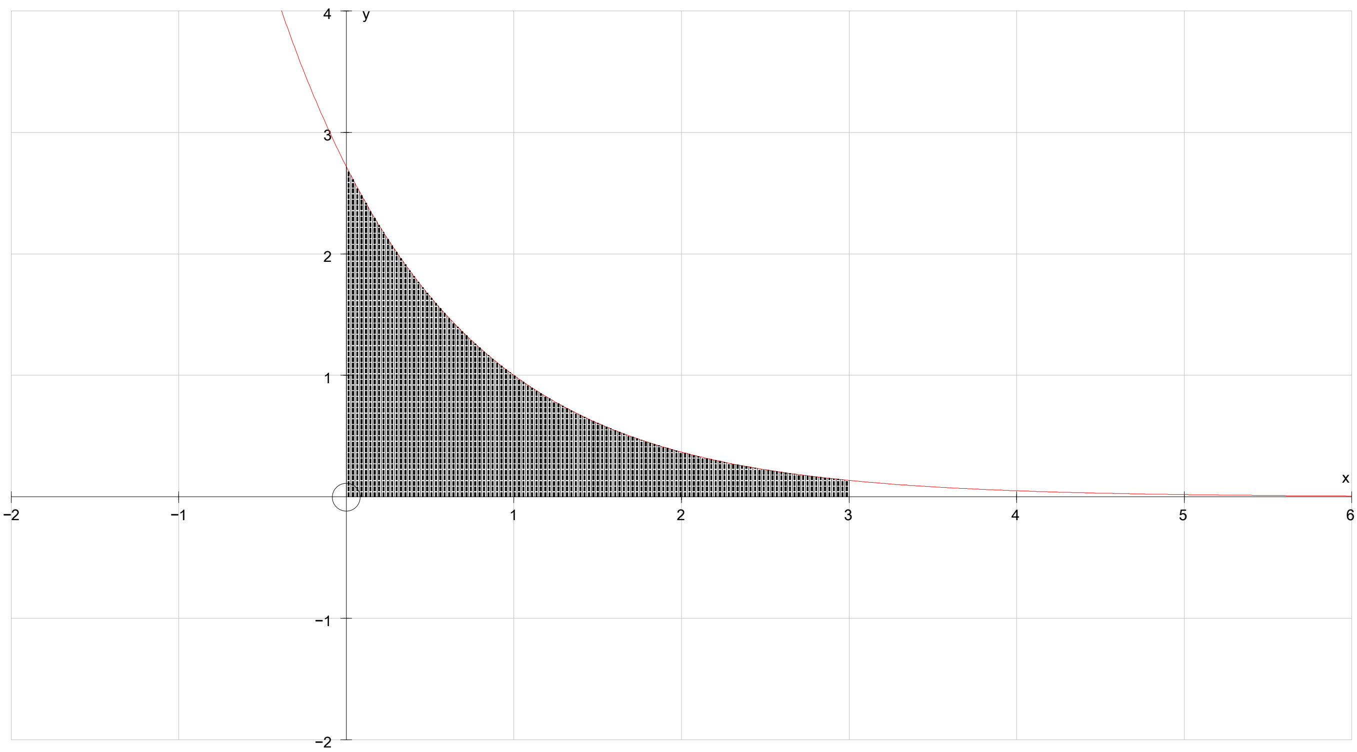
Just like the trapezium rule, the area under the curve is split up into strip, but instead of the tops of the strip being straight lines, they are parabola shaped.

Important : *n* must be EVEN therefore the number of ordinates *(y* values) will be ODD.

<u>Example</u>

Use Simpson's Rule, with 7 ordinates to find an approximation to

$$\int_0^3 e^{1-x}dx$$



$$\int_0^3 e^{1-x}\,dx$$

First we should tabulate the values of the ordinates

x values	y values
0	$y_0 = e^1$
0.5	$y_1 = e^{0.5}$
1	$y_2 = e^0$
1.5	$y_3 = e^{-0.5}$
2	$y_4 = e^{-1}$
2.5	$y_5 = e^{-1.5}$
3	$y_6 = e^{-2}$

With 6 strips : $I \approx \frac{0.5}{3}\left(y_0 + y_6 + 4(y_1 + y_3) + 2(y_2 + y_4)\right)$

$$I \approx \frac{0.5}{3}\left(e + \frac{1}{e^2} + 4(e^{0.5} + \frac{1}{e^{0.5}}) + 2(1 + \frac{1}{e})\right)$$

$$I \approx \frac{1}{6}(14.610...)$$

$I \approx 2.4$ (2sf)

Of course, the example above can be integrated in the familiar way.

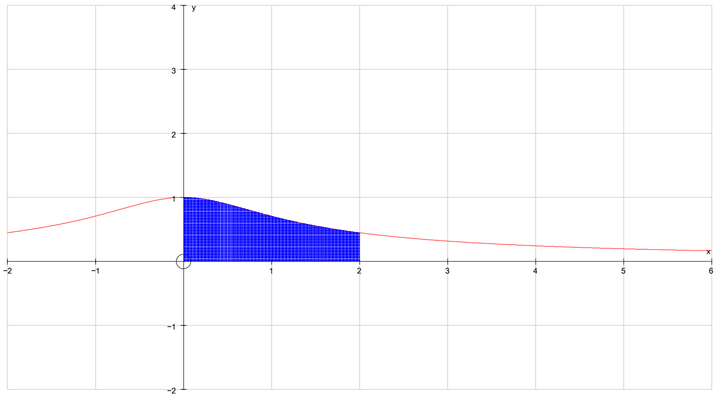
$$\int_0^3 e^{1-x}\,dx = \left[-e^{1-x}\right]_0^3 = \left(-e^{-2}\right) - \left(-e^{1}\right) = e - \frac{1}{e^2} = 2.583$$

Simpson's rule is used for integrating functions that cannot be integrated algebraically using C3 techniques we have learnt.

Example 2

Use simpson's rule with 8 intervals, to estimate the area bound by , $y = \dfrac{1}{\sqrt{x^2+1}}$ the

x axis and *x* = 2



NB : Autograph gives an area of 1.444

x values	*y values*
0	$y_0 = 1$
0.25	$y_1 = \dfrac{1}{\sqrt{1.0625}}$
0.5	$y_2 = \dfrac{1}{\sqrt{1.25}}$
0.75	$y_3 = \dfrac{1}{\sqrt{1.5625}}$
1	$y_4 = \dfrac{1}{\sqrt{2}}$
1.25	$y_5 = \dfrac{1}{\sqrt{2.5625}}$
1.5	$y_6 = \dfrac{1}{\sqrt{3.25}}$
1.75	$y_7 = \dfrac{1}{\sqrt{4.0625}}$
2	$y_8 = \dfrac{1}{\sqrt{5}}$

$$I \approx \frac{0.25}{3}\left(y_0 + y_8 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6)\right)$$

$$I \approx \frac{1}{12}\left(1 + \frac{1}{\sqrt{5}} + 4(bluecells) + 2(pinkcells)\right) \qquad I \approx \frac{1}{12}(17.32358788)$$

$$\boxed{I \approx 1.44}$$