VOLUMES of REVOLUTION

A volume of revolution is generated when an area is rotated about one of the coordinate axes.

The volume of the solid formed can be found using

$$V = \int_a^b \pi y^2 dx$$ or more easily used .... $$V = \pi \int_a^b y^2 dx$$

Taking π out as a factor often means answers will be in terms of π

This formula is used when an area is rotated 360° (or 2π radians) about the ***x*** axis.

Similarly, the volume of generated when an area is rotated about the *y* axis is found using

$$V = \pi \int_a^b x^2 dy$$

Problem 1

Find the volume generated when an area bound by the *x* and *y* axis, the line $x = 1$ and the curve $y = e^x$ is rotated through 360° about the *x* axis.

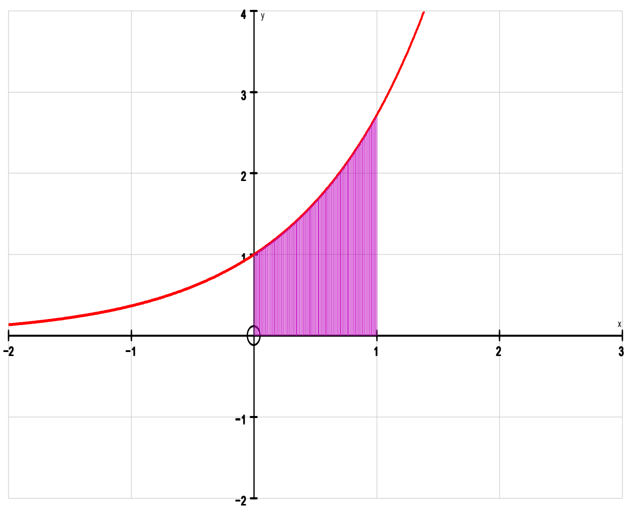


Diagram 1

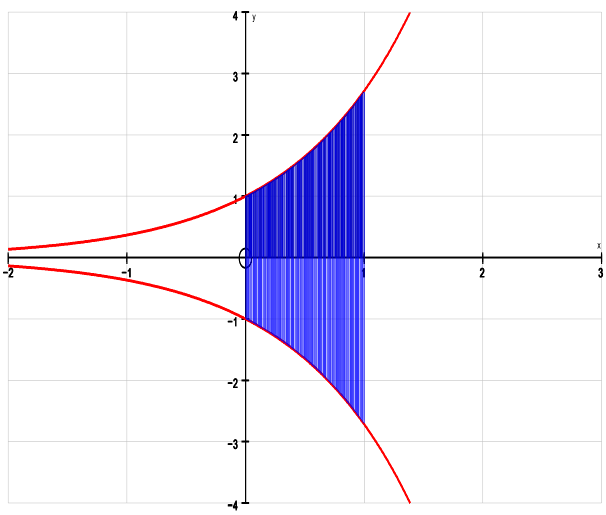


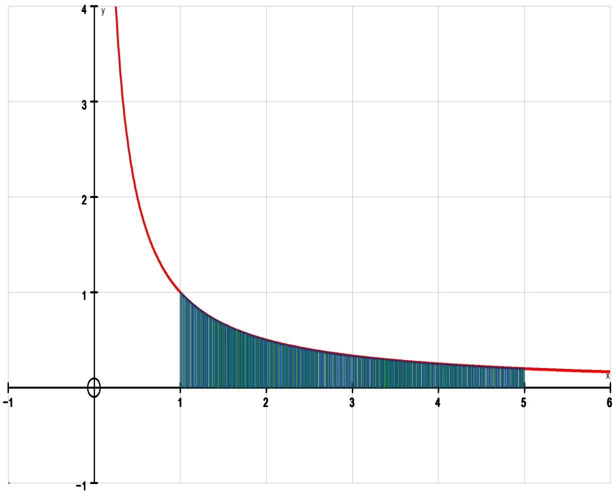
Diagram 2

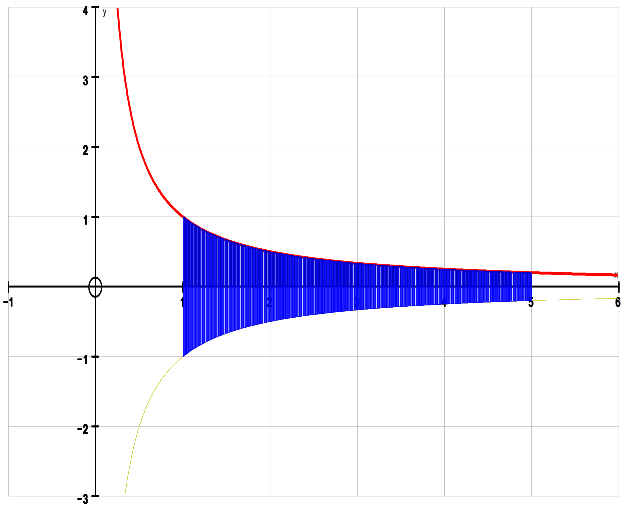
Notice how the area is diagram 1 becomes a volume in diagram 2.

Show your worked solution here

Problem 2

Calculate the volume generated when the area between the curve $y = \frac{1}{x}$ and the *x* axis from *x* = 1 to *x* = 5 is rotated through 2π about the *x* axis.

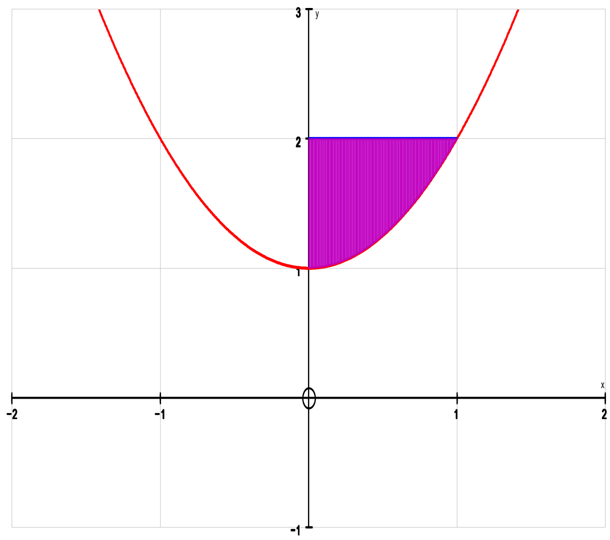


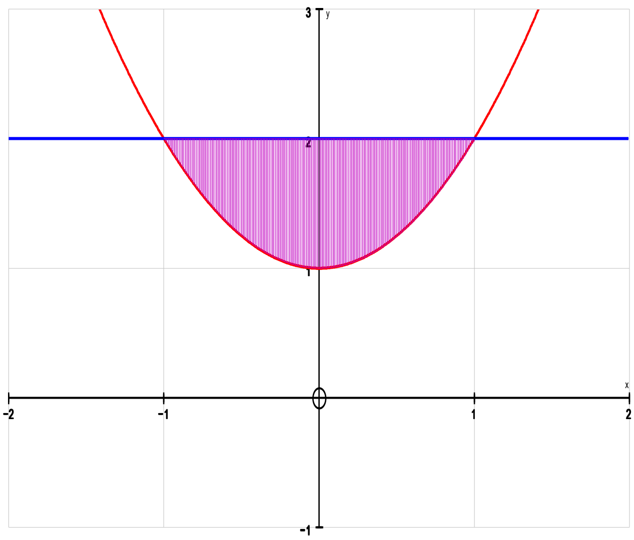


Worked solution

Problem 3 (Rotation around the *y* axis)

An area is bound by the inequalties $y \geq x^2+1$, $x \geq 0$ and $y \leq 2$. Find the volume generated when this area is completely rotated about the *y* axis.



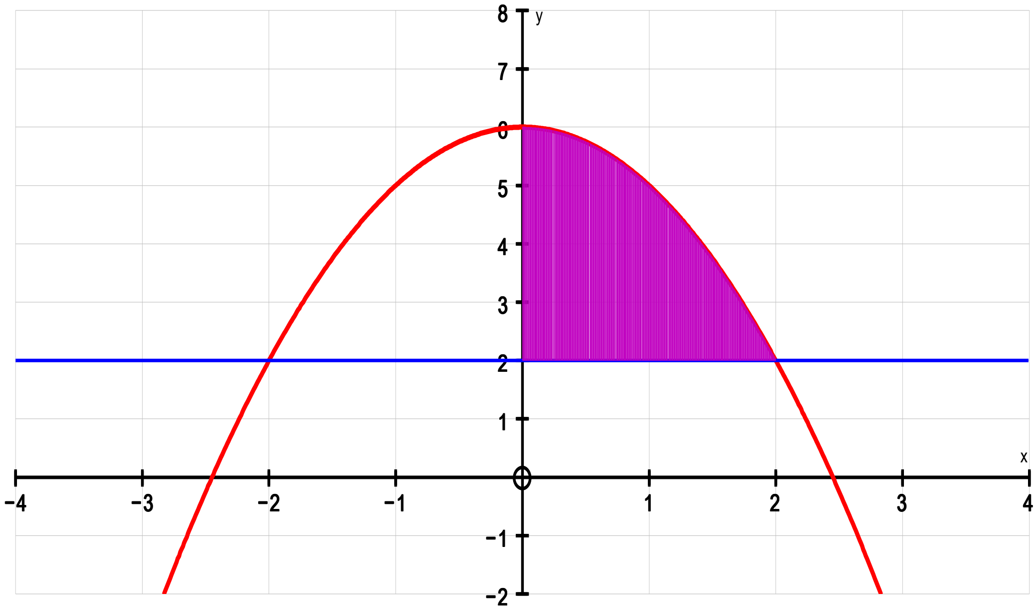


This time, we will need to use the other formula as we are integrating w.r.t. *y* (against the *y* axis)

Worked solution

Problem 4

(i) Find the area of the region in the first quadrant bounded by the *y*-axis and the line *y* = 2 and the curve $y = 6 - x^2$.



Working

(ii) Find the volume generated when this area is rotated 360° about the ***y* axis**

Working

Problem 5

The area bound by the curve $y = 4 - x^2$ and the line $y = 0$ is rotated fully about the y axis, to form a solid. Find the volume generated.

Sketch the graph here first, then solve using $V = \pi \int_a^b x^2 \, dy$

Problem 6

The area bound by the curve $y = x^3$, $y = 1$ and $y = 2$ for $x \geq 0$ is rotated fully about the y axis, to form a solid. Find the volume generated.

Sketch the graph here first, then solve using $V = \pi \int_a^b x^2 \, dy$

Problem 7

The area bound by the curve $y = \ln x$, $x = 0$, $y = 0$ and $y = 1$ is rotated fully about the y axis, to form a solid. Find the volume generated.

Sketch the graph here first, then solve using $V = \pi\int_a^b x^2\,dy$

Short Exercise

1 (a) Find the area of the region in the first quadrant bounded by the y axis, the line $y = 6$ and the curve $y = x^2 + 2$

(b) If this area is rotated completely about the y axis to form a solid, find the volume generated.

2. An area is bound by the line $y = 1$, the x axis and parts of the curve $y = 3 - x^2$. Find the volume generated when this area is rotated completely about the y axis.

Extension

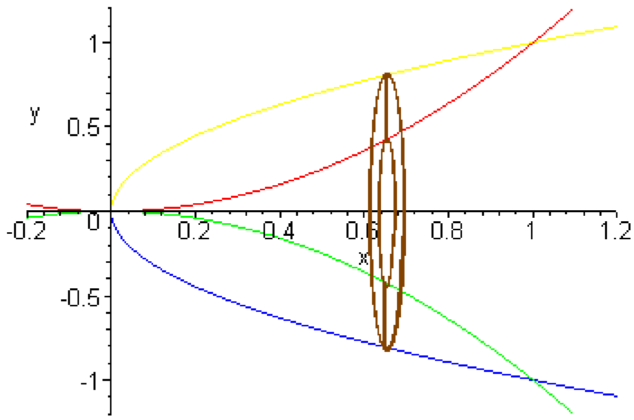
3. The area enclosed between the curves $y = x^2$ and $y^2 = x$ is rotated about the x axis. Find the volume generated.

Additional notes VOLUME by WASHERS

Example: Find the volume of the solid formed be revolving the region between the curves

$y = x^2$ and $y = \sqrt{x}$ about the x-axis.

Solution We draw the picture and revolve a cross section about the x-axis and come up with a washer.



The area of the Washer is equal to the area of the outer disk minus the area of the inner disk.

$$A = \pi(R^2 - r^2)$$

We have that R is the y-coordinate of the top curve ($y = \sqrt{x}$) and r is the y-coordinate of the bottom curve ($y = x^2$). We have

$A = \pi(\sqrt{x}^2 - [x^2]^2) = \pi[x - x^4]$ Hence :

$$\text{Volume} = \int_0^1 \pi\left(x - x^4\right)\,dx = \pi\left(\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$$

Volume by Washers

If the region below $y = f(x)$, above the $y = g(x)$, and between the lines $x = a$ and $x = b$ is revolved around the x-axis, then the volume of the resulting solid is given by

$$\text{Volume} = \pi\int_a^b (f(x))^2 - (g(x))^2\,dx$$