C4 Review 1

1. A curve is given parametrically by the equations

$$x = t(1+t) \qquad y = t^2(1+t)$$

(i) Find $\frac{dy}{dx}$ in terms of t.

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(ii) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$

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(iii) By first simplifying $\frac{y}{x}$, show that the curve has Cartesian equation $x^3 = xy + y^2$

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2. Given the equation of a circle is

$x^2 + y^2 - 2x + 6y - 15 = 0$, find $\frac{dy}{dx}$.

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3. Given that $\frac{dy}{dx} = (x \ln x) y^{\frac{1}{2}}$

And that *y* = 1 when *x* = 1, show that the value of *y* when *x* = 3 is $(\frac{9}{4} \ln 3)^2$

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4(i) Use the derivative of cos *x* to prove that

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

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(ii) Use the substitution *u* = sec *x* to find the exact value of

$$\int_0^{\frac{\pi}{3}} \sec^3 x \tan^3 x dx$$

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Answers

1. (i) $\frac{dy}{dx} = \frac{3t^2 + 2t}{2t + 1}$ (ii) $16x - 5y - 36 = 0$ (iii) $\frac{y}{x} = t$ (seen)

2. $$2x + 2y\left(\frac{dy}{dx}\right) - 2 + 6\left(\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2 - 2x}{2y + 6}$$

3. $\int y^{-\frac{1}{2}} dy = \int x \ln x dx$ Use of 'parts' for RHS.

Constant 'c' = 2.25

4. (i) Use of sec x as $\frac{1}{\cos x}$ to derive reciprocal function.

(ii) Correct method of substitution (inc. change of limits) to obtain $\boxed{3\frac{13}{15}}$