

## COMPLETING THE SQUARE

To carry out the process of completing the square for a quadratic polynomial :  
 Quadratics of the form  $ax^2 + bx + c$  can be reduced to the form  $a(x - p)^2 + q$  .  
 One advantage of this is to be able to sketch the graph of the quadratic.

### Example 1 for discussion

Let  $f(x) = x^2 + 6x - 7$

$$f(x) = [(x + 3)^2 - 9] - 7$$

$$f(x) = (x + 3)^2 - 16$$

### Example 2 for discussion

Let  $f(x) = 2x^2 - 12x + 20$

$$f(x) = 2[x^2 - 6x + 10]$$

$$f(x) = 2[(x - 3)^2 - 9 + 10]$$

$$f(x) = 2[(x - 3)^2 + 1] = 2(x - 3)^2 + 2$$

Now try to show the following quadratics in the form  $a(x - p)^2 + q$

1.  $x^2 + 5x - 3$

2.  $2x^2 - 8x - 5$

### Short exercise for practice

Solve these quadratics by completing the square first. Check your answers by using graphical methods or by substitution.

(a)  $x^2 - 4x - 21 = 0$

(b)  $x^2 - 3x + 2 = 0$

(c)  $x^2 - 2x - 2 = 0$

(d)  $x^2 - x - 5 = 0$

(e)  $x^2 - 5x + 10 = 0$

(f)  $(x - 2)(x - 3) = 0$

As mentioned earlier, one advantage of using this technique on a quadratic is to determine an important characteristic of the graph.

Consider the quadratic equation

$$x^2 - 6x + 5 = 0$$

The graph of  $y = x^2 - 6x + 5$  we know will have roots at  $x = 1, x = 5$

But we also can determine two other key points

(a) Where the curve cuts the y axis

(b) The value of x such that the curve is at a minimum (the y value)

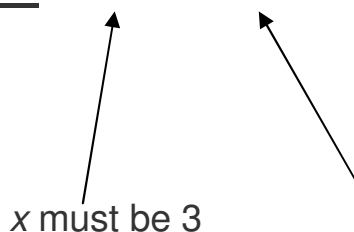
Let  $y = x^2 - 6x + 5$

$$y = (x - 3)^2 - 9 + 5$$

$$y = (x - 3)^2 - 4$$

when x = 3

so the minimum value of y = -4



to produce minimum value of -4

The general form will look like this

$$a(x + p)^2 + q$$

1. Determine the minimum value of  $x^2+3x-2$  and the value of  $x$  where this occurs.

2. (a) Solve the equation  $x^2-x-3=0$  by method of completing the square

(b) Determine the turning point (or vertex) on the graph  $y=x^2-x-3$