COMPLETING THE SQUARE

To carry out the process of completing the square for a quadratic polynomial :
Quadratics of the form $ax^2 + bx + c$ can be reduced to the form $a(x - p)^2 + q$.
One advantage of this is to be able to sketch the graph of the quadratic.

Example 1 for discussion

Let $f(x) = x^2 + 6x - 7$

$$f(x) = \left[(x+3)^2 - 9\right] - 7$$

$$f(x) = (x+3)^2 - 16$$

Example 2 for discussion

Let $f(x) = 2x^2 - 12x + 20$

$$f(x) = 2[x^2 - 6x + 10]$$

$$f(x) = 2[(x-3)^2 - 9 + 10]$$

$$f(x) = 2[(x-3)^2 + 1] = 2(x-3)^2 + 2$$

Now try to show the following quadratics in the form $a(x - p)^2 + q$

1. $x^2 + 5x - 3$
2. $2x^2 - 8x - 5$

Short exercise for practice

Solve these quadratics by completing the square first. Check your answers by using graphical methods or by substitution.

(a) $x^2 - 4x - 21 = 0$	(b) $x^2 - 3x + 2 = 0$
(c) $x^2 - 2x - 2 = 0$	(d) $x^2 - x - 5 = 0$
(e) $x^2 - 5x + 10 = 0$	(f) $(x-2)(x-3) = 0$

As mentioned earlier, one advantage of using this technique on a quadratic is to determine an important characteristic of the graph.

Consider the quadratic equation $\boxed{x^2 - 6x + 5 = 0}$

The graph of $y = x^2 - 6x + 5$ we know will have roots at $x = 1, x = 5$

But we also can determine two other key points
(a) Where the curve cuts the y axis
(b) The value of x such that the curve is at a minimum (the y value)

Let $y = x^2 - 6x + 5$

$$y = (x-3)^2 - 9 + 5$$

$$y = (x-3)^2 - 4$$

so the minimum value of *y* = –4 when x = 3

x must be 3

to produce minimum value of – 4

The general form will look like this $a(x+p)^2 + q$

1. Determine the minimum value of x^2+3x-2 and the value of *x* where this occurs.

2. (a) Solve the equation $x^2-x-3=0$ by method of completing the square

(b) Determine the turning point (or vertex) on the graph $y=x^2-x-3$