CO-ORDINATE GEOMETRY

Key points

You must be able to:

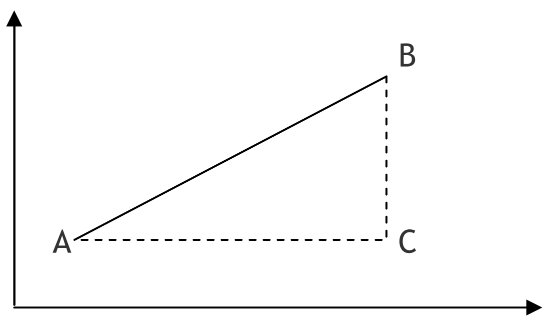
- Find the distance between two points
- Find the midpoint of a line segment
- Calculate the gradient of a line segment
- Find the equation of a line through a given point and gradient
- Find the equation of a line joining two points
- Use different forms of a straight line equation
- Find points of intersection of lines
- Use gradients to form equations of lines that are perpendicular (or parallel) to a given line.

Distance between 2 points

This is found by applying Pythagoras' Theorem

Example for discussion

A is the point (3, 2) and B is the point (7, 4)



Working

Generally, the distance between 2 given points (x_1, y_1) and (x_2, y_2) will be :

The midpoint of a line segment

Consider the previous line segment. The midpoint M is found by considering the midpoint of AC and BC on the diagram.

Midpoint of AC will be $\frac{x_1 + x_2}{2} = \frac{3+7}{2} = 5$

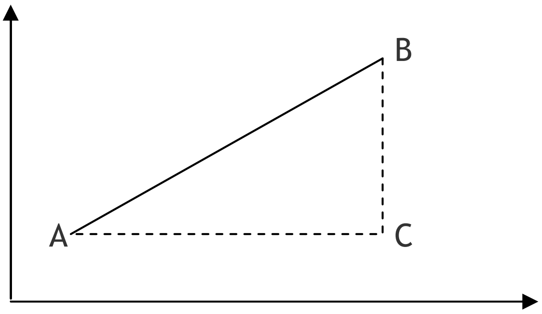
Midpoint of BC will be

So the midpoint of the line will be (______ , ______)

Generally, the midpoint of any line segment joining (x_1, y_1) and (x_2, y_2) will be

Calculating the gradient of a line segment

Example for discussion
A is the point (3, 2) and B is the point (7, 4)



Working:

Generally, the gradient of a line segment joining (x_1, y_1) and (x_2, y_2) will be

$m =$

Example 1
Find the gradient of the line that runs through the points A (−3, 1) and B (6, 4)

Working :

Example 2
Find the equation in the form $ax + by + c = 0$ of the line joining the points (−5, 2) and (3, −4).

Step 1 : Find the gradient
Step 2: Use a known point on the line along with the gradient to form an equation in the form $y = mx + c$. Find the intercept value of c.
Step 3: Rearrange the equation into the correct form.

Working :

Other useful method:

Let (x_1, y_1) be a point on the line. Let (x, y) be ANY point on the line.
The equation of the line will be

$$y - y_1 = m(x - x_1)$$

Find the equation of the line segment from A(3, 6) to B(- 3, 2)

Questions for discussion

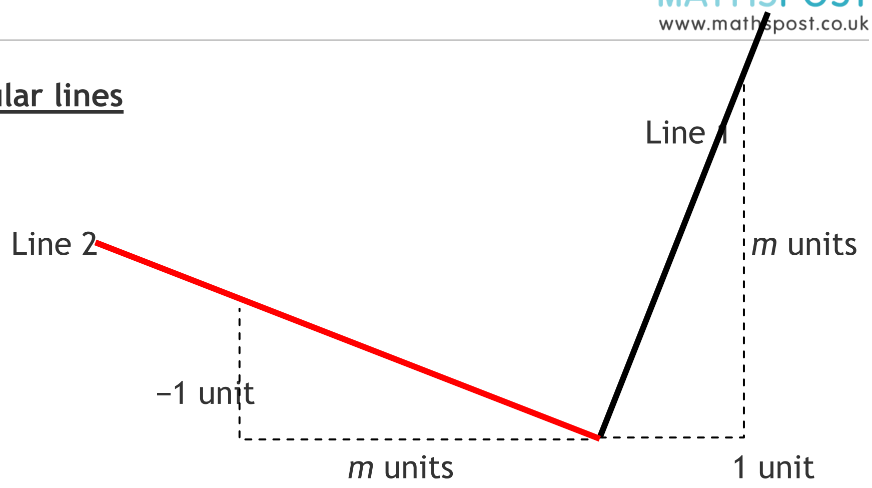
1. Find the equation of the straight line that passes through the points (3, −1) and (−2, 2). Hence find the coordinates of the point of intersection of the line and the *x* axis.

Working :

2. The coordinates of A and B are (3, 2) and (4, −5) respectively. Find the coordinates of the midpoint of AB and the gradient of AB.

Working :

The gradients of perpendicular lines



By definition, line 1 has a gradient of m. Line 2, which is perpendicular to line 1 has a gradient of $\frac{-1}{m}$

Therefore if a line l_1 has a gradient of m_1, a line perpendicular will have a gradient $m_2 = \frac{-1}{m_1}$

Moreover, the product of the two gradients will ALWAYS be equal to −1.

$$m_1 m_2 = -1$$

Example 3
Show that the line segment A(−1, 1) to B(1, 5) is perpendicular to the line segment P(−2, 3) to Q(2, 1)

Working :

Example 4
The line l_1 is the line segment A(3, - 2) to B(- 5, 4). Find the midpoint of l_1 and hence find the equation of the line l_2 which is the perpendicular bisector of the line l_1.

Working :

CO-ORDINATE GEOMETRY exercise

1. Find the length and gradient of the straight lines, joining the following pairs of points

(a) (4, 6) and (9, 15) (b) (5, –11) and (–1, 3) (c) (–2 ½ , – ½) and (4 ½ , –1)

2. Find whether AB is parallel or perpendicular to PQ in the following cases ;

(a)	A (4, 3)	B (8, 4)	P (7, 1)	Q (6, 5)
(b)	A (–2, 0)	B (1, 9)	P (2, 5)	Q (6, 17)
(c)	A (8, –5)	B (11, –3)	P (1, 1)	Q (–3, 7)
(d)	A (–6, –1)	B (–6, 3)	P (2, 0)	Q (2, –5)

3. A (–3, 1) B (1, 2) C (0, –1) D (–4, –2) are the vertices of a parallelogram.
(a) Find the lengths of the sides AB and AD
(b) Find the midpoint of BC.

4. A rectangle has vertices P (1, 7) Q (7, 5) R (6, 2) S (0, 4)
(a) Find the length of the diagonals
(b) Find the point of intersection of the diagonals

5. Find the intercepts on the axes made by the straight line $3x - 2y + 10 = 0$.
Hence find the area of the triangle enclosed by the axes and this line.

6. Write down the equations of the straight lines joining the following points, in the form $y = mx + c$

(a)	(–1, 1) and (1, –1)	(b)	(–4, 2) and (8, 4)
(c)	(4, –2) and (6, –1)	(d)	(–3, 7) and (6, 1)

7. Write down the equation of the straight line which ;
(a) runs through (5, 11) parallel to the x axes.
(b) is the perpendicular bisector of the line joining (2, 0) and (6, 0)
(c) runs through (0, –10) parallel to $y = 6x + 3$
(d) runs through (0, –1) perpendicular to $3x - 2y + 5 = 0$

8. Find the equation of the straight line joining the origin to the mid-point of the line joining A (3, 2) and B (5, –1)

Exam style questions

1. The points A and B have coordinates (1, 5) and (3, 1) respectively. Find the equation of the perpendicular bisector of the line AB.

2. Determine the gradient of the straight line $2x-3y+9=0$. Find also, the equation of the straight line through the origin which is perpendicular to the line $2x-3y+9=0$

3. A straight line *l* has equation $x+3y=14$. Find the gradient of this line.

The point A (a, 2a) lies on *l*. Find the value of a.

Using the value of a, find the equation of the straight line that passes through A and is perpendicular to *l*. Give your answer in the form $y=mx+c$

4. The straight line *p* passes through the point (10, 1) and is perpendicular to the line *r* with equation $2x+y=1$. Find the equation of *p*.

Find also the coordinates of the point of intersection of *p* and *r* and deduce the perpendicular distance from the point (10, 1) to the line *r*.

5. The equation of a straight line l_1 is $x+3y-33=0$. The point P is (3, 0) and the point Q is (6, 9). The straight line l_2 is parallel to l_1 and passes through P.

(i) Find the equation of l_2 giving your answer in the form *ax* + *by* + c = 0
(ii) Verify the Q lies on l_1.
(iii) Show that the line joining P and Q is perpendicular to l_1.
(iv) Find the perpendicular distance between l_1 and l_2.

6. Find the equation of the straight line which is parallel to the line $x+4y-1=0$ and which passes through the point of intersection of the lines $y=2x$ and $x+y-3=0$

Answers

1. $x-2y+4=0$
2. gradient = $\frac{2}{3}$ $\quad y=\frac{-3}{2}x$
3. gradient = $\frac{-1}{3}$ $\quad a=\frac{14}{5}$ $\quad 15x-5y-14=0$
4. $x-2y-8=0$ $\quad$ (2, - 3) $\quad 4\sqrt{5}$
5. (i) $x+3y-3=0$
   (iv) $3\sqrt{10}$
6. $x+4y-9=0$