Key ideas for DIFFERENTIATION in C1

Learn how to apply this key rule to a range of functions

$$If\ y = x^n\ then\ \frac{dy}{dx} = nx^{n-1}$$

Find $\frac{dy}{dx}$ for each of the following :

(a) $y = x^3 - x^7$

$\frac{dy}{dx} =$

(b) $y = 2x^4 - 3x$

$\frac{dy}{dx} =$

(c) $y = 2x^3 + 4x - 3$

$\frac{dy}{dx} =$

(d) $y = \sqrt{x}$

$\frac{dy}{dx} =$

(e) $y = \frac{1}{x} - \frac{2}{x^2}$

$\frac{dy}{dx} =$

(f) $y = \frac{2+x}{\sqrt{x}}$

$\frac{dy}{dx} =$

You may need to find the value of the gradient (the derivative) of the curve at a particular point.

Example to work through….

Find the value of $\frac{dy}{dx}$ at the point where $x = 3$ on the curve whose equation is

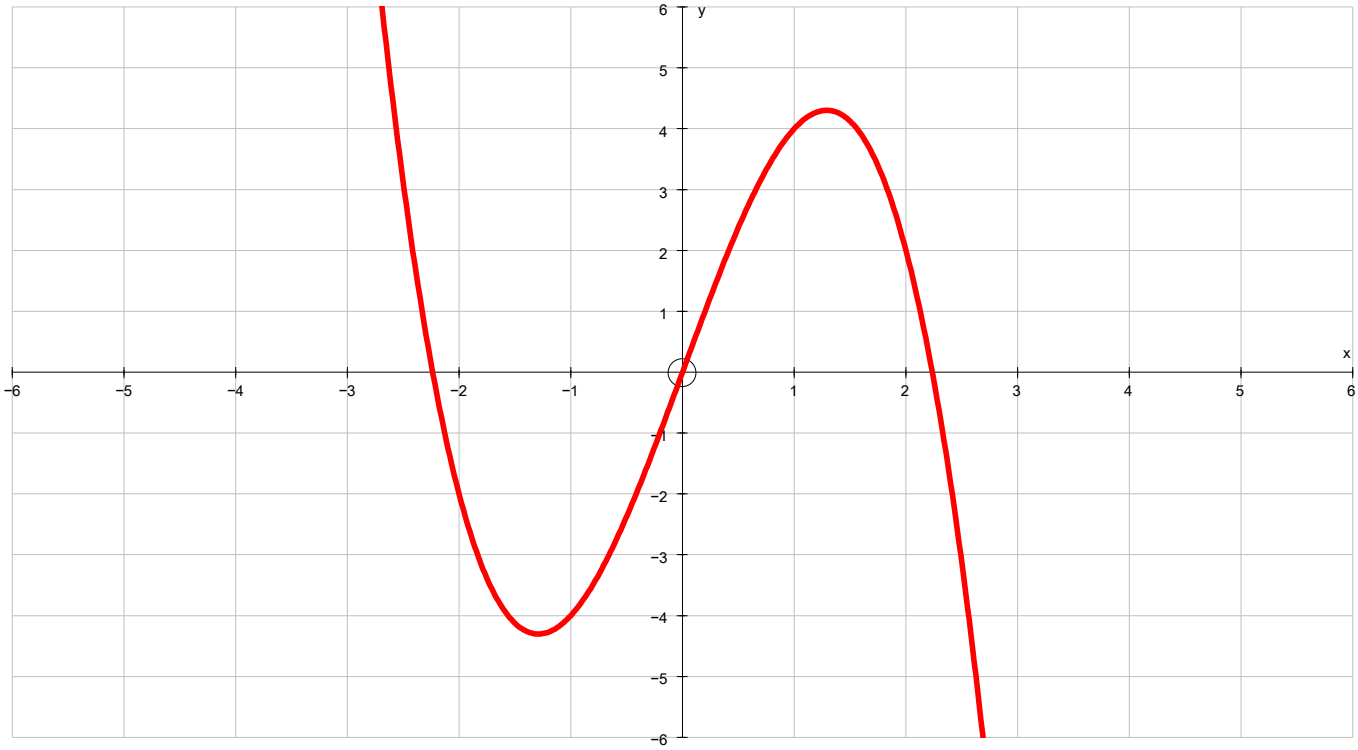
$y = (2x-1)(3x+2)$

You may be given the value of the derivative (the steepness of the tangent) and be asked to locate the tangent to the curve – this is usually in the form of a coordinate but sometimes just the *x* value.

Example 2 to work through….

For the curve $y = 5x - x^3$, determine the coordinates on the curve where the gradient is – 7 .

I have plotted the curve for you to interpret your answer. Use your answer to draw tangents on the diagram.



Working :

Tangents and Normals

Now we are able to calculate the gradient of a tangent (through a given point on the curve) we can now begin to find the equation of this tangent and beyond this, employ some earlier coordinate geometry knowledge to determine the equation of the Normal to the tangent (through this point).

Example 3 to work through….

(a) Find the equation of **the tangent** to the curve $y = (2x-3)^2$ at the point $x = 2$.

(b) Find the equation of the Normal to the curve $y = \frac{2}{x}$ at the point $x = \frac{3}{2}$

(c) Find the equation of the tangent to the curve $y = x^2 - 2x$ which is perpendicular to the line $2y = x - 1$

Alternative notation to $\frac{dy}{dx}$:

$f(x)$ = function $\quad$ $f'(x)$ = derivative $\quad$ $f''(x)$ = second derivative

Questions to work through……

Find the derived function, $f'(x)$, for each of the following :

1. $f(x) = 3x^4 - 2x^3 + x^2 - x + 10$ $\quad$ $f'(x) =$

2. $f(x) = 2x(3x^2 - 4)$ $\quad$ $f'(x) =$

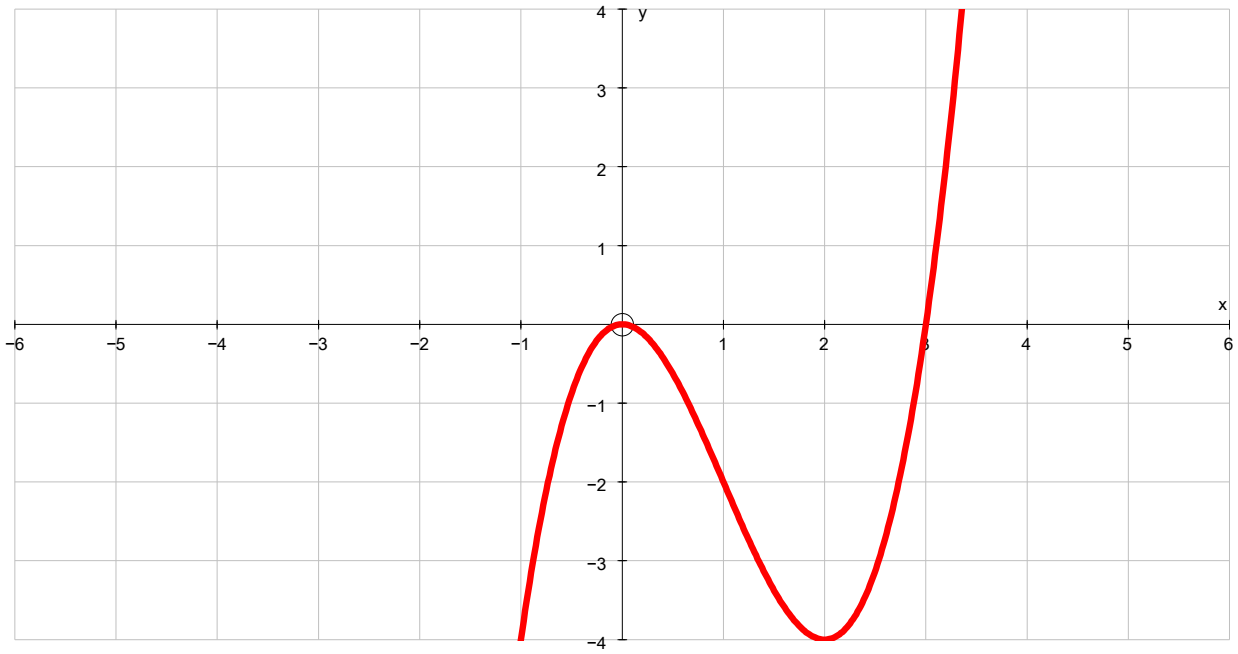
3. $f(x) = \frac{x^5 + 3x^3}{x^2}$ $\quad$ $f'(x) =$

4. $f(x) = (4x - 5)^2(3 - 2x)$ $\quad$ $f'(x) =$

5. $f(x) = ax^3 + \frac{x}{b} + cx - d$ $\quad$ $f'(x) =$

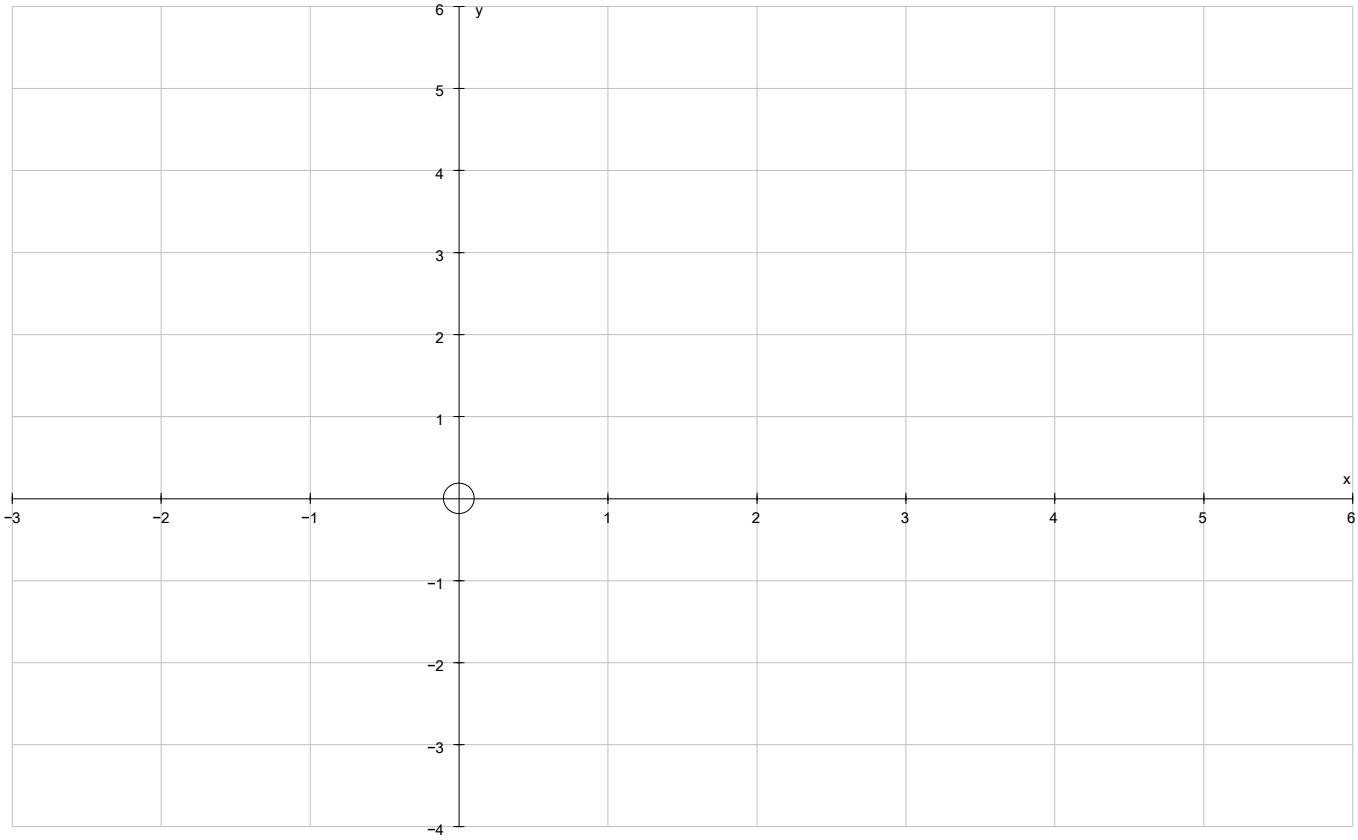
6. (i)Find the gradient of the curve $y = 9x - x^3$ at the point where ***x*** = 1.
(ii) Find the equation of the tangent to the curve at this point. Where does the tangent meet the line ***y*** = ***x*** ?

7. The graph shows the curve $y = x^2(x-3)$. Find the equation of the tangent and the normal to the curve $y = x^2(x-3)$ at the point where the curve cuts the *x* axis.



Working:

8. Repeat question 7 for the curve $y = x(x-4)^2$. Sketch the curve on the axis below.



Working : (Hint – consider where the repeated root is in this equation)

You may wish to use this page for additional notes