RECOGNISING A QUADRATIC IN SOME FUNCTION OF *x*

By use of some simple substitution, polynomials of a higher order can be written as quadratics in order to solve equations.

Example 1

Solve $x^4 - 5x^2 + 4 = 0$

Let $y = x^2$ thus $x^4 - 5x^2 + 4 = 0$

Becomes $y^2 - 5y + 4 = 0$

$y^2 = (x^2)^2$

Factorise in *y* $(y-4)(y-1) = 0$

Solve for y $y = 4$ or $y = 1$ Now $x = \sqrt{y}$

Therefore **$x = \pm 2$ or ± 1**

Example 2

Solve $x^{-4} - 5x^{-2} + 4 = 0$

Let $y = x^{-2}$ thus $x^4 - 5x^2 + 4 = 0$

Becomes $y^2 - 5y + 4 = 0$

$y^2 = (x^{-2})^2$

Factorise in *y* $(y-4)(y-1) = 0$

Solve for y $y = 4$ or $y = 1$ Now $x = \frac{1}{\sqrt{y}}$

Therefore $x = \pm\frac{1}{2} \, or \pm 1$

Example 3

Of course not all 'disguised' quadratics will have integer powers. Look at the example below:
You may be asked to consider rational powers….

Solve $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4 = 0$

Let $y = x^{\frac{1}{3}}$ thus $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4 = 0$

Becomes $y^2 - 5y + 4 = 0$

$y^2 = \left(x^{\frac{1}{3}}\right)^2$

Factorise in *y* $(y-4)(y-1) = 0$

Solve for y $y = 4$ or $y = 1$

Now $x = y^3$

Therefore ***x* = 64 or 1**

Short exercise for practice

Solve the following equations making clear the substitution you are using.

(a) $x^4 + 2x^2 - 8 = 0$

(b) $2x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 3 = 0$

(c) $9x^4 + 8x^2 - 1 = 0$

NOTE : A quartic equation will have at most 4 real roots. The example in part (c) has only 2 real roots.