

## Practice Exam Questions : Proof by induction

Prove by induction that :

$$(a) \quad \sum_{r=1}^n r^2(r-1) = \frac{1}{12}n(n-1)(n+1)(3n+2)$$

$$(b) \quad \text{If } u_{n+1} = 3u_n + 4 \quad \text{and } u_1 = 2 \quad \text{then, } u_n = 4 \times 3^{n-1} - 2$$

$$(c) \quad \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$$

$$(d) \quad \text{Let } A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} \quad \text{Prove by induction that } A^n = \begin{pmatrix} 1 & 0 \\ 1-2^n & 2^n \end{pmatrix}$$