COMPLEX NUMBERS PRACTICE TEST 1

1. In an argand diagram, the point P represents the complex number z. On a single diagram, illustrate the set of possible positions of P for each of the cases

(a) $|z - 3i| < 3$ (b) $\arg(z + 3 - 3i) = \frac{\pi}{4}$

Given that z satisfies both (a) and (b), find the greatest possible value of $|z|$.

2. Given that $|z - 5| = |z - 2 - 3i|$, show on an argand diagram the locus of the point which represents z. Use your diagram to show that there is no value of z satisfying both

$|z - 5| = |z - 2 - 3i|$ and $\arg z = \frac{\pi}{4}$

3. The quadratic equation $z^2 + 6z + 34 = 0$ has complex roots α and β.

(a) Find the two roots, in the form a + bi

(b) Find the modulus and argument of each root, and illustrate the two roots on an argand diagram.

(c) Find $|\alpha - \beta|$

4. Prove by induction that if $u_{n+1} = u_n + 2$ and $u_1 = 3$ then $u_n = 2n + 1$

5. Find the determinant of AB given that $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 0 & -2 \end{pmatrix}$

6. A parallelogram with one vertex at the origin has an area of 8. It is transformed by the matrix A where $A = \begin{pmatrix} 4 & 2 \\ -4 & 3 \end{pmatrix}$. Write down the area of the image parallelogram.

7. (a) Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$

(b) Hence find the sum of the first n terms of the series

$$\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} \dots + \frac{2r+1}{r^2.(r+1)^2}$$

(c) Show that the series is convergent and state the sum to infinity