IMPLICIT DIFFERENTIATION

So far, differentiation has been concerned with functions that are **explicit**. These are functions in the form y = f(x) such as $y = 2x^3 + 3x - 4$

In C4, we must also consider functions defined **implicitly s**uch as the circle $\mathbf{x^2 + y^2 = 25}$

Clearly if we can differentiate these functions we can find equations of tangents and normals etc..

Consider differentiating each term separately as before but for terms in 'y' , we must apply the chain rule

See why here….differentiating y **with respect to x**

$$\frac{d}{dx}(y) = \frac{d(y)}{dy} \times \frac{dy}{dx}$$

So, for example, the derivative of y^2 with respect to x : $\frac{d}{dx}(y^2) = 2y \times \frac{dy}{dx}$

Our circle equation $\mathbf{x^2 + y^2 = 25}$

Derivative :

Make $\frac{dy}{dx}$ the subject to find our 'gradient formula'

Try these for size….

Differentiate w.r.t. x

$x^3 + y^2 + 3x - 4y = 1$

Find the gradient of the tangent at the point (x_1, y_1) to the curve with equation $x^2 - 2y^2 - 6y = 0$

Using the **PRODUCT RULE**

Differentiate w.r.t. x $x^3 + \mathbf{xy^2} - y^3 = 5$

Find dy/dx if $2x + xe^y = 5$

Implicit differentiation - review work

1. Find an expression for $\frac{dy}{dx}$ in terms of *x* and *y* for ;

(a) $3x^2 - 6xy + 2y^2 = 5$

(b) $e^x y - e^y x = 1$

2. Find the equation of the normal at the point (1, 1) to the curve given by ;

$$9x^2 y + 6xy^2 = 15$$

3. Find the equation of the tangent and the normal at the point (2, 1) to the curve given by ;

$$3x^2 y^3 - x^3 y^2 = 4x$$

Answers

1. (a) $\frac{dy}{dx} = \frac{3y - 3x}{2y - 3x}$

(b) $\frac{dy}{dx} = \frac{e^y - e^x y}{e^x - e^y x}$

2. Differentiating wrt *x*; $18xy + 9x^2 \frac{dy}{dx} + 6y^2 + 12xy\frac{dy}{dx} = 0$

Equation of normal ; 8y = 7x + 1

3. Differentiating wrt *x*; $6xy^3 + 9x^2 y^2 \frac{dy}{dx} - 3x^2 y^2 - 2x^3 y\frac{dy}{dx} = 4$

Equations : $5y = x + 3$ and $y + 5x = 11$