# PARAMETRIC EQUATIONS

Sometimes the equation of a curve is given by expressing the coordinates $x$ and $y$ as functions of a third variable (usually $t$), called a parameter.

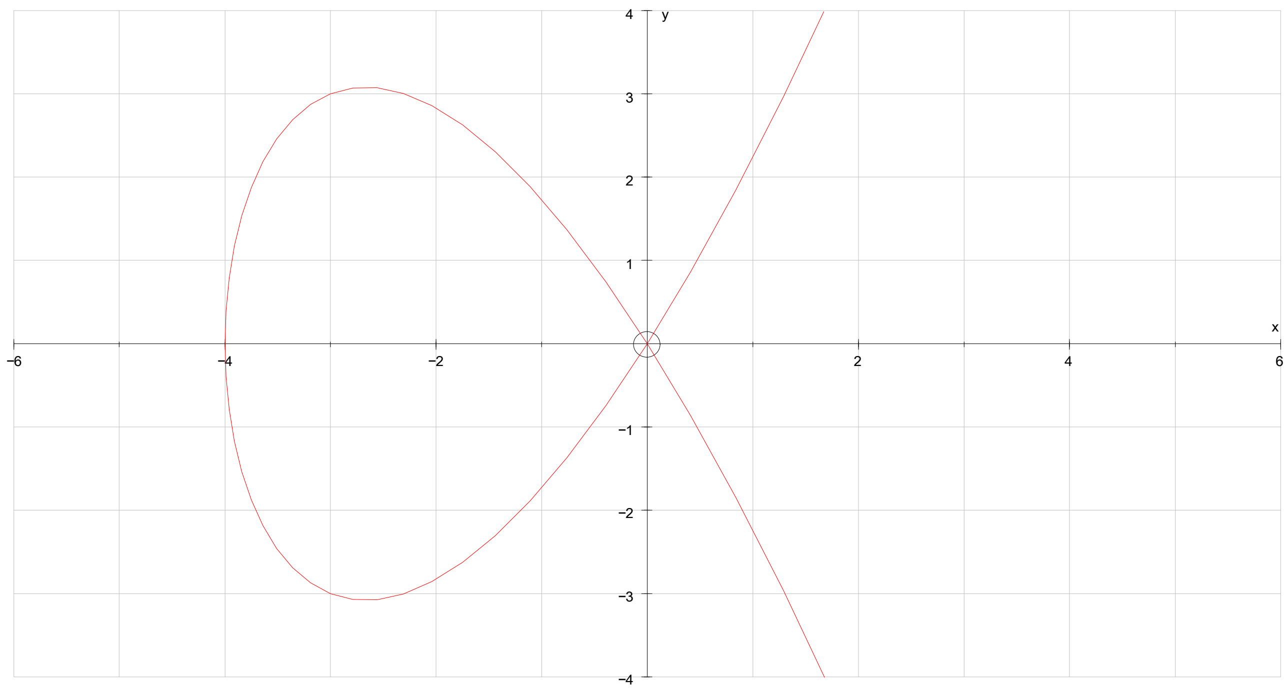
Using $t$ as parameter enables us to refer to a particular point on quite complex curves (that we've not met so far)

## Example 1

Plot the curve given parametrically by the equations

$x = t^2 - 4$ and $y = t^3 - 4t$ *use t from* $-3$ to 3

| $t$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $t^2 - 4$ | | | | | | | |
| $t^3 - 4t$ | | | | | | | |




What is the Cartesian equation connecting $x$ and $y$ ?

Use substitution by making $t$ the subject of either parametric equation

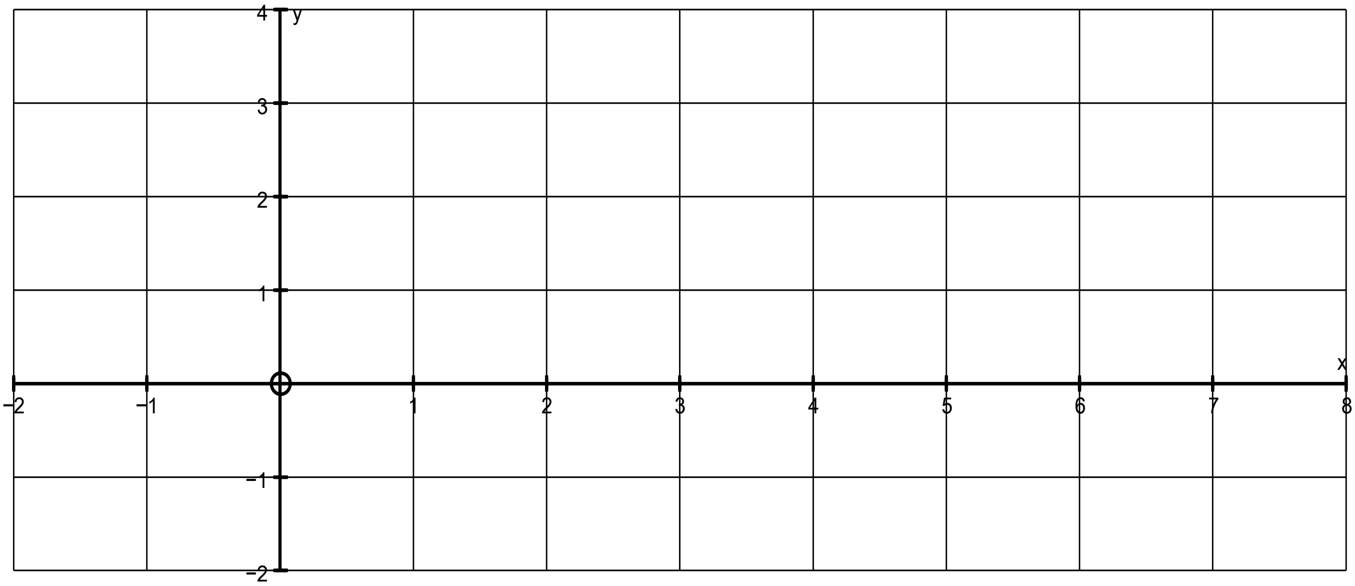
| Working : |
|---|
| |

So the Cartesian equation reads $\boxed{y^2 = x^2(x+4)}$

## Some curves to plot

1. $x = t^2 + 1$ and $y = t + 2$ *use t from* −3 to 3

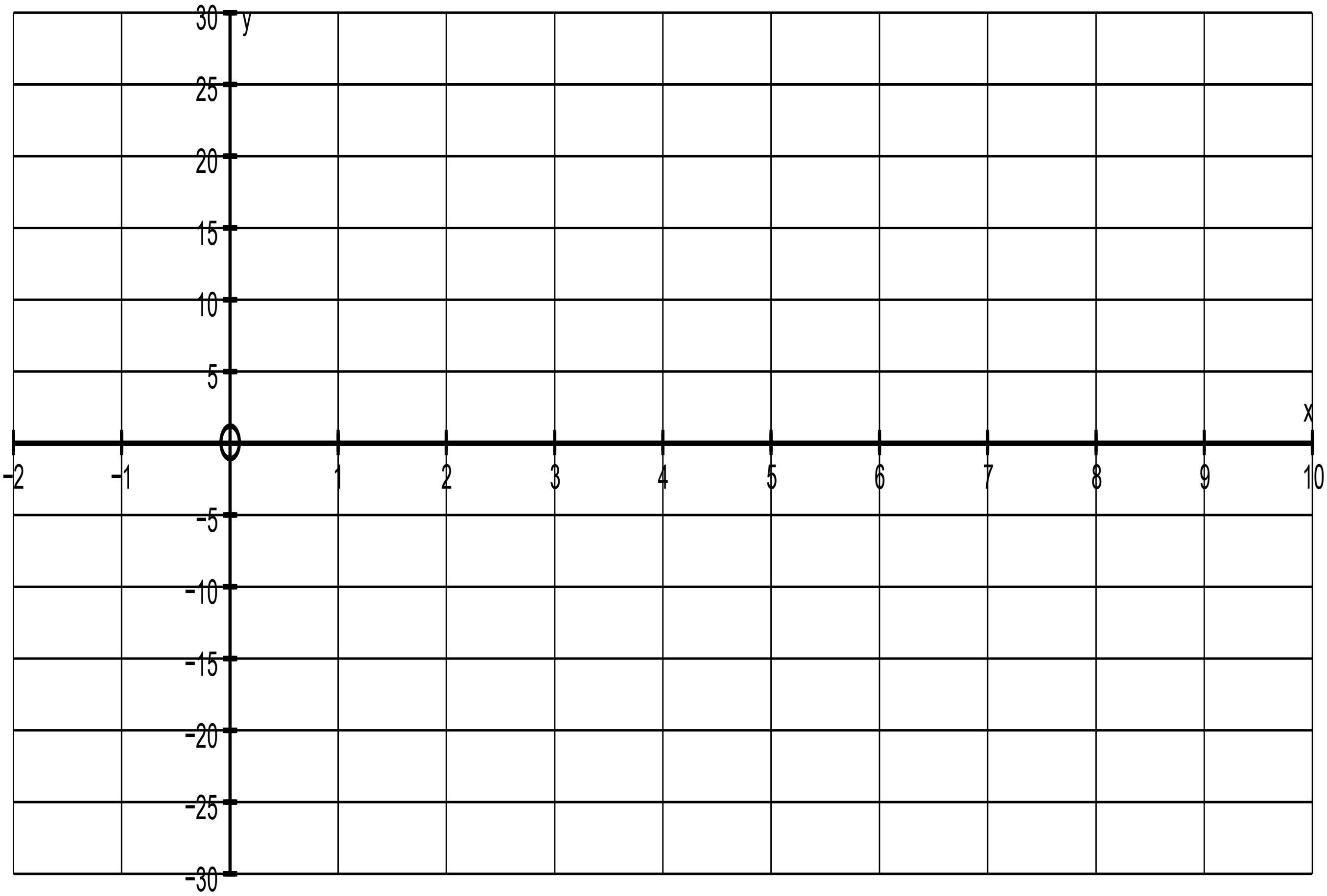
| $t$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $t^2 + 1$ | | | | | | | |
| $t + 2$ | | | | | | | |




***Find the Cartesian equation of the loci above***

2. $x = t^2$ and $y = t^3$ *use t from* −3 to 3

| $t$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $t^2$ | | | | | | | |
| $t^3$ | | | | | | | |




3. Find the value of the parameter and the other coordinate on the curve

$x = t$ and $y = \frac{2}{t}$ at $y = \frac{3}{2}$

[2]

4. Show that the parametric equations

(i) $x = 1 + 2t$ and $y = 2 + 3t$

(ii) $x = \frac{1}{2t-3}$ and $y = \frac{t}{2t-3}$

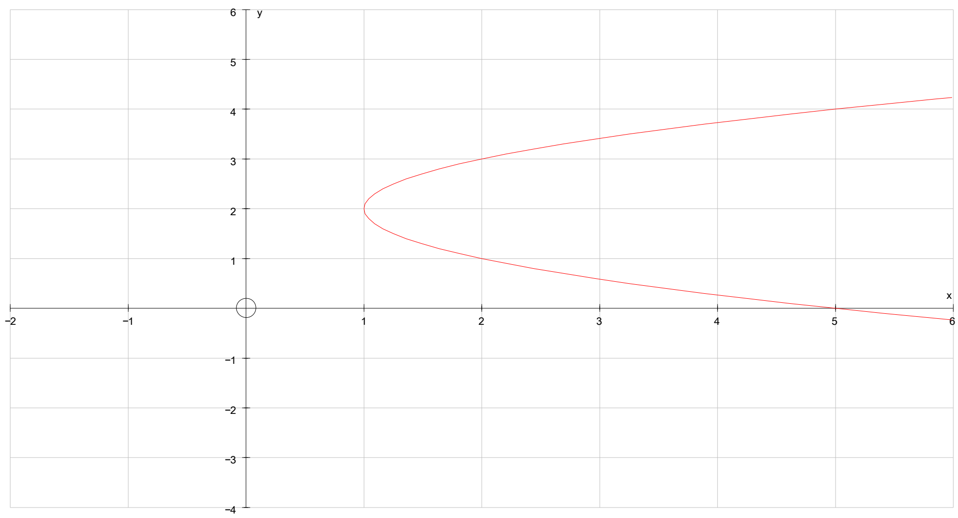
[6]

Both represent the same straight line. (Form the same cartesian equations)

## Answers

1. $x = t^2 + 1$ and $y = t + 2$ *use t from* −3 to 3

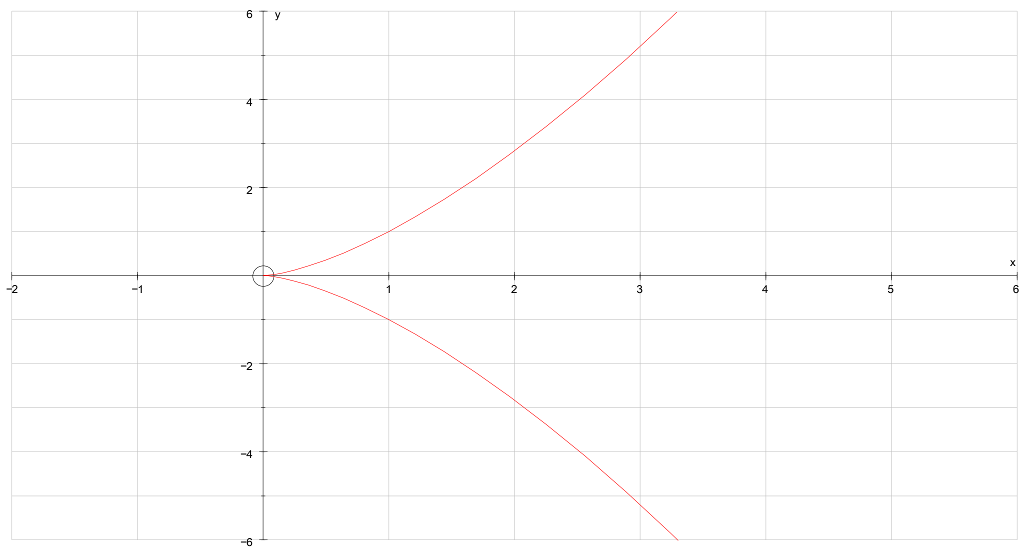
| $t$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $t^2 + 1$ | 10 | 5 | 2 | 1 | 2 | 5 | 10 |
| $t + 2$ | −1 | 0 | 1 | 2 | 3 | 4 | 5 |



***Cartesian equation :*** $x = y^2 - 4y + 5$

2. $x = t^2$ and $y = t^3$ *use t from* −3 to 3

| $t$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $t^2$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |
| $t^3$ | - 27 | -8 | - 1 | 0 | 1 | 8 | 27 |



***Cartesian equation :*** $y^2 = x^3$

3. $t = x = \frac{4}{3}$

4. Both should give the Cartesian equation **3*x* - 2*y* + 1= 0**

## PARAMETRIC DIFFERENTIATION - SOME EXAMPLES

| $x = f(t)$ | $y = g(t)$ | $\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$ |
|---|---|---|

Find the derivative $\frac{dy}{dx}$ for each of the following parametric equations,

1. $x = t^2, y = 2t$

2. $x = t^2, y = t^3$

3. $x = \sin t, y = \cos 2t$

4. $x = e^t, y = te^t$

5. $x = \sin^3\theta, y = \cos^3\theta$

Answers:

1. $\frac{1}{t}$ 2. $\frac{3t}{2}$ 3. $-4\sin t$ 4. $1 + t$ 5. $-\cot\theta$

## Further examples to work through

6. Find an expression for $\frac{dy}{dx}$ for each of the parametrically defined curves ;

(a) $x = 4t^2 - 1, y = t^3 + t$

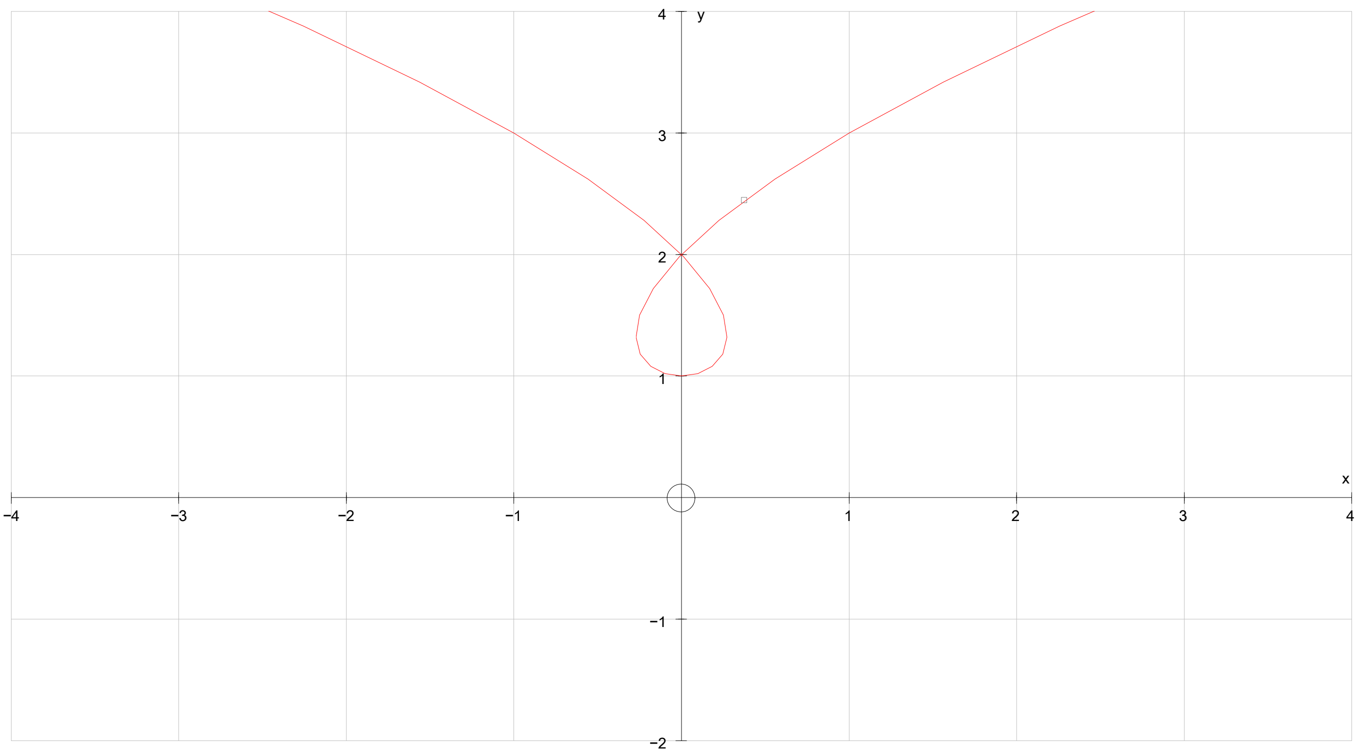
(b) $x = 3\sin\theta, y = \cos 2\theta$

7. Find the gradient at $t = 2$, on the parabola $x = t^2$, $y = 2t$

8. Find the coordinates of the points on the parametrically defined curve where the gradient has a value of 1. Give each answer using 2dp.

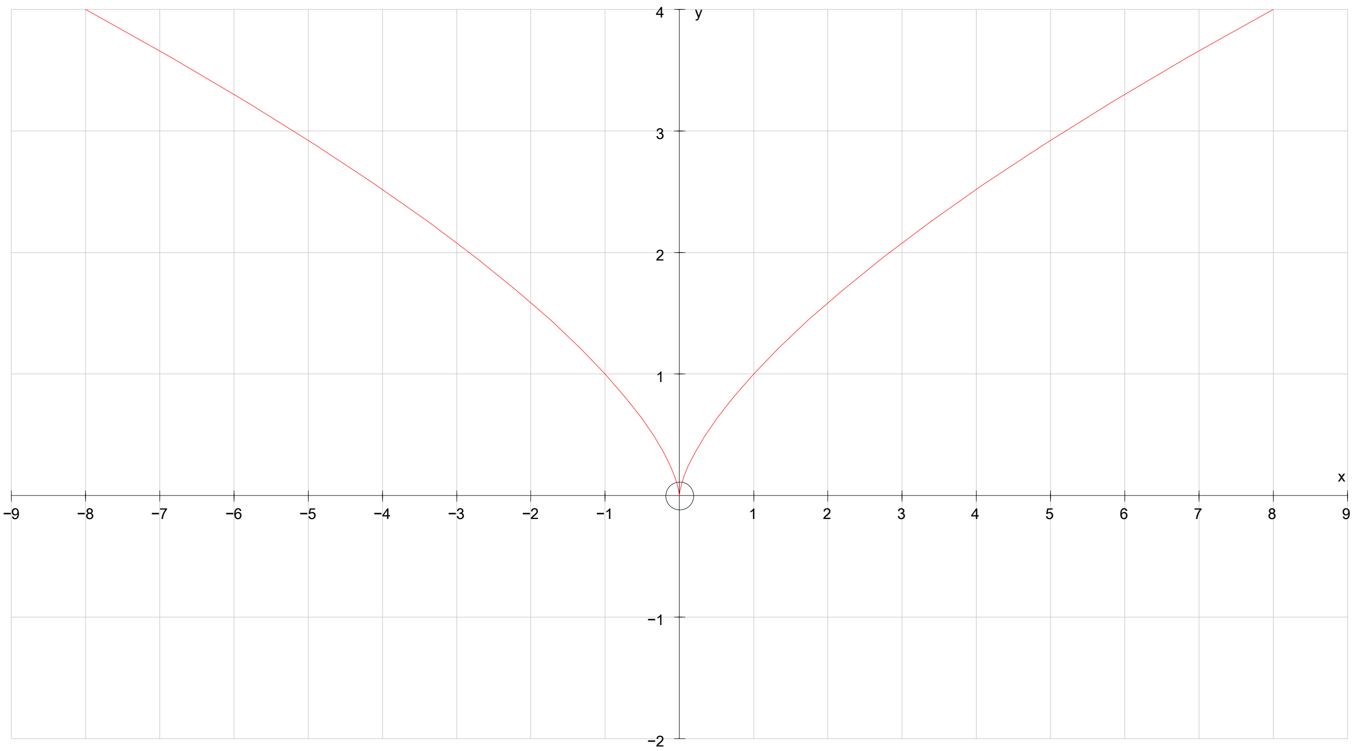
$$x = 2t^3 - t \qquad y = 2t^2 + 1$$

I have included the curve for you to interpret the solution graphically.



9. Find the equation of the normal at ( –8,  4) to the curve which is given parametrically by $x = t^3$ , $y = t^2$ .

Draw the normal on the curve below. Label the normal with the straight line equation.




Note : The gradient is not defined when $t$ = 0 because the tangent at the origin is the $y$ axis. The point where $t$ = 0 is called a **cusp**.