THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For discussion

Use the quadratic formula to try to solve the following equations;

1. $x^2 - 10 + 24 = 0$
2. $x^2 - 10 + 25 = 0$
3. $x^2 - 10 + 26 = 0$

What do you notice about your results?
Try to explain why some quadratics have two solutions, some have only one and some have none.

Try to make up some quadratic equations that have two solutions, only one solution or no solutions.

How can you tell if the graph is a tangent to the *x* axis without sketching the graph?

PROPERTIES OF QUADRATIC ROOTS

Recall that to solve a quadratic equation :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The sum of the roots

The separate roots are (when written in a different way) :

$x = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$

When the roots are added together, the terms containing the roots disappear leaving $\frac{-b}{a}$ This is a good quick check on an accurate root calculation.

The discriminant $b^2 - 4ac$

The quadratic formula has actually two terms. It is the second term which needs to be carefully considered when determining the NATURE of the roots.

Consider 3 cases:

1. If $b^2 - 4ac > 0$ then its square root will a real number (whole, decimal or fraction) In this case, we say the equation has 2 distinct, real roots.

2. If $b^2 - 4ac = 0$ then the square root is also zero. Therefore there is just one value of *x* that satisfies the equation. This root is said to be a repeated root.

3. If $b^2 - 4ac < 0$ We cannot find its square root. (It is negative!) In this case, the equation has no real roots.

Can the following quadratic equations be solved? If so, how many solutions do they have?

$x^2 - 2x + 1 = 0$ $\qquad$ $x^2 - 2x - 5 = 0$ $\qquad$ $x^2 - 2x + 3 = 0$

Short exercise

1. Find the number of distinct real roots of the given equations :

(a) $3x^2 - 2x - 5 = 0$

(b) $x^2 + 4x + 8 = 0$

2. Find the values of k for which $2(k-1)x^2 + 2kx + k - 1 = 0$ has **equal** roots.

3. Find the range of values of k such that the given equation **has two roots**.

(a) $2x^2 - 3x + k = 0$ $\qquad$ (b) $kx^2 - 4x + 2 = 0$